11

IMPULSE AND MOMENTUM

Conceptual Questions

11.1. The velocities and masses vary from object to object, so there is no choice but to compute $p_x = mv_x$ for each one and then compare:

$$
\begin{aligned}
p_{1x} &= (20\text{ g})(1\text{ m/s}) = 20\text{ g m/s}\\
p_{2x} &= (20\text{ g})(2\text{ m/s}) = 40\text{ g m/s}\\
p_{3x} &= (10\text{ g})(2\text{ m/s}) = 20\text{ g m/s}\\
p_{4x} &= (10\text{ g})(1\text{ m/s}) = 10\text{ g m/s}\\
p_{5x} &= (200\text{ g})(0.1\text{ m/s}) = 20\text{ g m/s}
\end{aligned}
$$

So the answer is $p_{2x} > p_{1x} = p_{3x} = p_{5x} > p_{4x}$.

11.2. The impulse is equal to the change in momentum, so

$$\Delta p_x = mv_{\text{fx}} - mv_{\text{ix}} = 4\text{ Ns}$$

The final velocity is thus

$$v_{\text{fx}} = v_{\text{ix}} + \frac{4\text{ Ns}}{m} = 1\text{ m/s} + \frac{4\text{ Ns}}{2\text{ kg}} = 3\text{ m/s}$$

Since the velocity is positive, the object is moving to the right.

11.3. The impulse is equal to the change in momentum, so

$$\Delta p_x = mv_{\text{fx}} - mv_{\text{ix}} = -4\text{ Ns}$$

The final velocity is thus

$$v_{\text{fx}} = v_{\text{ix}} - \frac{4\text{ Ns}}{m} = 1\text{ m/s} - \frac{4\text{ Ns}}{2\text{ kg}} = -1\text{ m/s}$$

Since the velocity is negative, the object is now moving to the left with a speed of 1 m/s. Note that the impulse was negative, which decreases the initially positive velocity.

11.4. When the question talks about forces, times, and momenta, we immediately think of the impulse-momentum theorem, which tells us that, to change the momentum of an object, we must exert a net external force on it over a time interval: $\Delta\vec{p} = \vec{F}_{\text{avg}}\Delta t$. Because equal forces are exerted over equal times, the impulses are equal and the changes in momentum are equal. Because both carts start from rest, the change in momentum of each is the same as the final momentum of each, so their final momenta are equal. Notice that, to answer the question, we do not need to know the mass of either cart, or even the specific time interval (as long as it is the same for both carts).

11.5. The impulse-momentum theory tells us that the change in momentum of an object is related to the net force on the object and the length of time the force was applied. Mathematically, $\Delta\vec{p} = \vec{F}_{\text{avg}}\Delta t.$ The same force applied to the two carts results in a larger acceleration for the less massive plastic cart (Newton's second law), enabling it to travel the 1-m distance in a shorter time. Therefore, the plastic cart has a smaller change in momentum than the lead cart. Because the final momentum of each cart is equal to their change in momentum (zero initial momentum), the final momentum of the plastic cart is less than that of the lead cart.

11.6. In this story, Carlos is correct. During the short time of the bullet-block collision other forces are negligible compared to the force between the bullet and block, so in the impulse approximation momentum is conserved. When the bullet bounces off of the steel block, the bullet's final momentum is backward. To balance that, the steel block must be moving forward faster than the case in which the bullet embeds itself in the wooden block (in which case the bullet has a final momentum in the forward direction).

11.7. The impulse-momentum theory states that a change in an object's momentum results when a net force is applied to the object for some time interval; $\Delta\vec{p} = \vec{F}_{\text{avg}}\Delta t.$ Stopping a hard ball requires changing its momentum, and this change can be accomplished with a small force over a long time interval or a large force over a short time interval. The padding in a glove lets the time interval during which the ball is stopped be long, resulting in a smaller force on the glove and on your hand.

11.8. The impulse-momentum theory states that a change in an object's momentum results when a net force is applied to the object for some time interval; $\Delta\vec{p} = \vec{F}_{\text{avg}}\Delta t.$ Stopping an automobile requires changing its momentum from some to none. This change can be accomplished with a small force over a long time interval or a large force over a short time interval. The crumple zone that collapses during an automobile collision lengthens the time interval during which the automobile is stopped, resulting in a smaller force on the passengers as they also come to a stop.

11.9. The club and ball form a system. The interaction force when the club and ball collide is very large compared to other forces at the time of collision, such as gravity and the force of the golfer on the club. So, in this impulse approximation, momentum is conserved during the collision. After the club hits the ball, it will give the ball some of its momentum. The club can continue to move forward as long as the momentum the ball obtains is less than the initial momentum of the club. Note that the momentum conservation is valid only if we consider the short time between just before and just after the collision. The wider we make the time window, the more time gravity and the golfer have to influence the motion of the club and ball, so that momentum conservation would no longer hold for the club-ball system.

11.10. The impulse one ball receives is equal to the average force on it from the other ball multiplied by the time during which the force is applied. But by Newton's third law the force that the rubber ball exerts on the steel ball is equal to the force the steel ball exerts on the rubber ball. So both balls receive the same amount of impulse, although the impulses are in opposite directions. This is true regardless of the masses of the balls.

11.11. The impulse one ball receives is equal to the average force on it from the other ball multiplied by the time during which the force is applied. But by Newton's third law the force that the rubber ball exerts on the steel ball is equal to the force the steel ball exerts on the rubber ball. So both balls receive the same amount of impulse, although the impulses are in opposite directions.

11.12. **(a)** Both particles cannot be at rest immediately after the collision. If they were both at rest, then the sum of the momenta after the collision would be zero, and since momentum is conserved in collisions, it would have had to be zero before as well (and it wasn't).
(b) If the masses are equal and the collision is elastic, the moving particle will stop and give all of its momentum to the previously resting particle. A good example of this appears when a billiard ball collides head-on with another billiard ball that is at rest.

We say momentum is conserved in all collisions because we assume that both colliding objects are part of the system and we assume the "impulse approximation" prevails, meaning that other forces can be neglected during the short time interval of the collision. In part **(a)**, if the system contained a third particle that participated in the collision, then it is possible for the first two particles to end up at rest if the momentum were carried off by the third particle.

11.13. **(a)** Let Paula and Ricardo be a system. Initially, their total momentum is zero since they are at rest. After they push off each other, the total momentum must still be zero, so Ricardo and Paula must have equal but opposite momenta.
(b) Momentum $p = mv.$ Since Paula is less massive than Ricardo, her speed must be higher than Ricardo's for her to have the same momentum as Ricardo.

11.14. The problem can be divided into three parts: (1) from when the first ball is released and to just before it hits the stationary ball, (2) the two balls collide, and (3) the two balls swing up together just after the collision to their highest point. Energy is conserved in parts (1) and (3) as the balls swing like pendulums, but during the collision in part (2) momentum is conserved but energy is not. So both energy and momentum conservation are each separately used as you work through each part of the problem.

Exercises and Problems

Exercises

Section 11.1 Momentum and Impulse

11.1. **Model:** Model the bicycle and its rider as a particle. Also model the car as a particle.
Solve: From the definition of momentum,

$$p_{\text{car}} = p_{\text{bicycle}} \quad \Rightarrow \quad m_{\text{car}} v_{\text{car}} = m_{\text{bicycle}} v_{\text{bicycle}} \quad \Rightarrow \quad v_{\text{bicycle}} = \left(\frac{m_{\text{car}}}{m_{\text{bicycle}}} \right) v_{\text{car}} = \left(\frac{1500 \text{ kg}}{100 \text{ kg}} \right) (5.0 \text{ m/s}) = 75 \text{ m/s}$$

Assess: This is a very high speed $(\approx 168 \text{ mph})$. This problem shows the importance of mass in comparing two momenta.

11.2. **Model:** Model the car and the baseball as particles.
Solve: **(a)** The momentum $p = mv = (3000 \text{ kg})(15 \text{ m/s}) = 4.5 \times 10^4$ kg m/s.
(b) The momentum $p = mv = (0.20 \text{ kg})(40 \text{ m/s}) = 8.0$ kg m/s.

11.3. **Solve:** The impulse J_x is defined as $J_x = \int_{t_i}^{t_f} F_x(t)dt =$ area under the $F_x(t)$ curve between t_i and t_f. The area under the force-time curve is

$$J_x = \tfrac{1}{2}(4.0 \text{ ms} - 0.0 \text{ ms})(1000 \text{ N}) + (2.0 \text{ ms})(1000 \text{ N}) = 4.0 \text{ N} \cdot \text{s}.$$

11.4. **Model:** The particle is subjected to an impulsive force.
Solve: The impulse is the area under the force-time curve. From 0 to 2 ms the impulse is zero, as with the interval between 8 ms and 12 ms. The only non-zero contribution to the impulse is between 2 ms and 8 ms.
From 2 to 8 ms the impulse is

$$\int F(t)dt = \tfrac{1}{2}(2000 \text{ N})(8.0 \text{ ms} - 2.0 \text{ ms}) = 6.0 \text{ Ns}$$

This is the entire impulse.

11.5. Solve: The impulse is defined as $J_x = \int_{t_\text{i}}^{t_\text{f}} F_x(t)dt =$ area under the $F_x(t)$ curve between t_i and t_f. For the force-time curve shown, the impulse is

$$\begin{aligned}6.0\text{ N}\cdot\text{s} &= \tfrac{1}{2}(F_\text{max})(2.0\text{ ms} - 0.0\text{ ms}) + F_\text{max}(6.0\text{ ms} - 2.0\text{ ms}) + \tfrac{1}{2}(F_\text{max})(8.0\text{ ms} - 6.0\text{ ms})\\ &= F_\text{max}(1.0\text{ ms} + 4.0\text{ ms} + 1.0\text{ ms}) \Rightarrow F_\text{max} = 1.0\times10^3\text{ N}\end{aligned}$$

11.6. Model: Model the object as a particle and the interaction as a collision.
Visualize: Please refer to Figure EX11.6.
Solve: The object is initially moving to the right (positive momentum) and ends up moving to the left (negative momentum). Using the impulse-momentum theorem $p_\text{fx} = p_\text{ix} + J_x$,

$$-2\text{ kg m/s} = +6\text{ kg m/s} + J_x \quad \Rightarrow \quad J_x = -8\text{ kg m/s} = -8\text{ Ns}$$

Since $J_x = F_\text{avg}\Delta t$, we have

$$F_\text{avg}\Delta t = -8\text{ Ns} \quad \Rightarrow \quad F_\text{avg} = \frac{-8\text{ Ns}}{10\text{ ms}} = -8\times10^2\text{ N}$$

Thus, the force is 800 N to the left.

11.7. Model: Model the object as a particle and the interaction as a collision.
Visualize: Please refer to Figure EX11.7.
Solve: The momentum bar chart tells us the final momentum $(p_\text{fx} = 2\text{ kg m/s})$ and the impulse $(J_x = 6\text{ kg m/s})$. Using the impulse-momentum theorem $p_\text{fx} = p_\text{ix} + J_x$, we can find the initial momentum:

$$p_\text{ix} = p_\text{fx} - J_x = 2\text{ kg m/s} - 6\text{ kg m/s} = -4\text{ kg m/s}$$

Since $p_\text{ix} = mv_\text{ix}$ we have $v_\text{ix} = p_\text{ix}/m = (-4\text{ kg m/s})/(0.05\text{ kg}) = -80\text{ m/s}$. The speed is thus 80 m/s and the direction is to the left.
11.8. Model: Model the object as a particle and the interaction with the force as a collision.
Solve: Using the equations

$$p_\text{fx} = p_\text{ix} + J_x \text{ and } J_x = \int_{t_\text{i}}^{t_\text{f}} F_x(t)dt = \text{area under force curve}$$

$$(2.0\text{ kg})v_\text{fx} = (2.0\text{ kg})(1.0\text{ m/s}) + (\text{area under the force curve})$$

$$v_\text{fx} = (1.0\text{ m/s}) + \tfrac{1}{2.0\text{ kg}}(1.0\text{ s})(-2.0\text{ N}) = 0.0\text{ m/s}$$

Assess: For an object with positive velocity, a positive impulse increases the object's speed. The opposite is true for an object with negative velocity.

11.9. Model: Model the object as a particle and the interaction with the force as a collision.
Solve: Using the equations

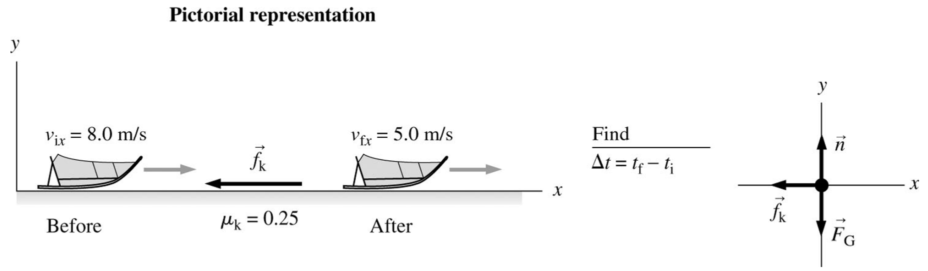
$$p_\text{fx} = p_\text{ix} + J_x \text{ and } J_x = \int_{t_\text{i}}^{t_\text{f}} F_x(t)dt = \text{area under force curve}$$

$$(2.0\text{ kg})v_\text{fx} = (2.0\text{ kg})(1.0\text{ m/s}) + (\text{area under the force curve})$$

$$v_\text{fx} = (1.0\text{ m/s}) + \tfrac{1}{2.0\text{ kg}}(8.0\text{ N})(0.50\text{ s}) = 3.0\text{ m/s}$$

Because v_fx is positive, the object is still moving to the right, but at 3.0 m/s.
Assess: For an object with positive velocity, a positive impulse increases the object's speed. The opposite is true for an object with negative velocity

11.10. Model: Use the particle model for the sled, the model of kinetic friction, and the impulse-momentum theorem.
Visualize:



Note that the force of kinetic friction f_k imparts a negative impulse to the sled.

Solve: Using $\Delta p_x = J_x$, we have

$$p_{fx} - p_{ix} = \int_{t_i}^{t_f} F_x(t)dt = -f_k \int_{t_i}^{t_f} dt = -f_k \Delta t \quad \Rightarrow \quad mv_{fx} - mv_{ix} = -\mu_k n \Delta t = -\mu_k mg \Delta t$$

We have used the model of kinetic friction $f_k = \mu_k n$, where μ_k is the coefficient of kinetic friction and n is the normal (contact) force by the surface. The force of kinetic friction is independent of time and was therefore taken out of the impulse integral. Thus,

$$\Delta t = \frac{1}{\mu_k g}(v_{ix} - v_{fx}) = \frac{(8.0 \text{ m/s} - 5.0 \text{ m/s})}{(0.25)(9.8 \text{ m/s}^2)} = 1.2 \text{ s}$$

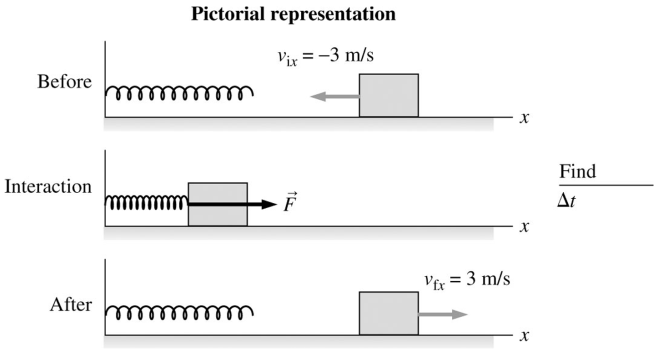
11.11. Model: Model the rocket as a particle, and use the impulse-momentum theorem. The only force acting on the rocket is due to its own thrust.
Visualize: Please refer to Figure EX11.11.
Solve: (a) The impulse is

$$J_x = \int F_x(t)dt = \text{area of the graph of } F_x(t) \text{ between } t = 0 \text{ s and } t = 30 \text{ s} = \tfrac{1}{2}(1000 \text{ N})(30 \text{ s}) = 1.5 \times 10^4 \text{ Ns}$$

(b) From the impulse-momentum theorem, $p_{fx} = p_{ix} + J_x(t)$, so the momentum or velocity increases as long as J_x is positive. If J_x becomes negative, the speed will stop increasing and start decreasing, so this point will be the maximum. For the current problem, the impulse is always positive, so the speed increases continuously. The maximum is therefore at the end of the impulse $(t = 30 \text{ s})$. The speed at this point is

$$mv_{fx} = mv_{ix} + 1.5 \times 10^4 \text{ Ns} \quad \Rightarrow \quad (425 \text{ kg})v_{fx} = (425 \text{ kg})(75 \text{ m/s}) + 1.5 \times 10^4 \text{ Ns} \quad \Rightarrow \quad v_{fx} = 110 \text{ m/s}$$

11.12. Model: Model the glider cart as a particle, and its interaction with the spring as a collision. Assume a frictionless track.
Visualize:



Solve: Using the impulse-momentum theorem $p_{fx} - p_{ix} = \int F(t)dt$,

$$(0.60\text{ kg})(3\text{ m/s}) - (0.60\text{ kg})(-3\text{ m/s}) = \text{area under force curve} = \tfrac{1}{2}(36\text{ N})(\Delta t) \quad \Rightarrow \quad \Delta t = 0.2\text{ s}$$

11.13. Model: Model the ball as a particle, and its interaction with the wall as a collision in the impulse approximation.
Visualize: Please refer to Figure EX11.13.
Solve: Using the equations

$$p_{fx} = p_{ix} + J_x \text{ and } J_x = \int_{t_i}^{t_f} F_x(t)dt = \text{area under force curve}$$

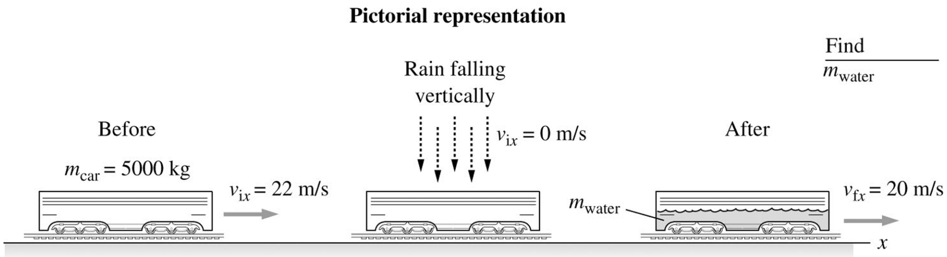
$$(0.250\text{ kg})v_{fx} = (0.250\text{ kg})(-10\text{ m/s}) + (500\text{ N})(8.0\text{ ms})$$

$$v_{fx} = (-10\text{ m/s}) + \left(\frac{4.0\text{ N}}{0.250\text{ kg}}\right) = 6.0\text{ m/s}$$

Assess: The ball's final velocity is positive, indicating it has turned around.

Section 11.2 Conservation of Momentum

11.14. Model: Choose car + rainwater to be the system.
Visualize:

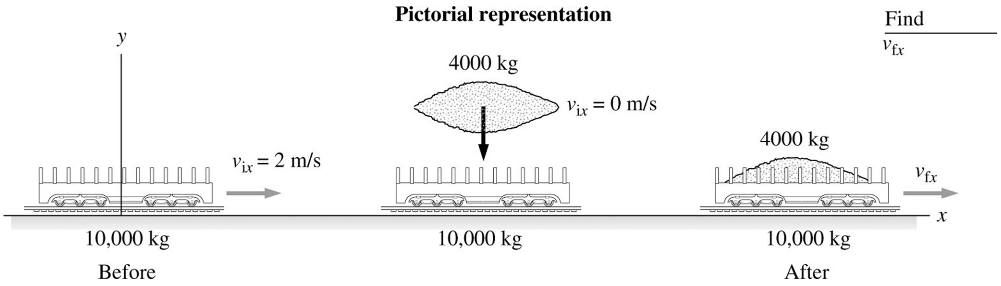


There are no *external* horizontal forces on the car + water system, so the horizontal momentum is conserved.
Solve: Conservation of momentum gives $p_{fx} = p_{ix}$. Hence,

$$(m_{car} + m_{water})(20\text{ m/s}) = (m_{car})(22\text{ m/s}) + (m_{water})(0\text{ m/s})$$

$$(5000\text{ kg} + m_{water})(20\text{ m/s}) = (5000\text{ kg})(22\text{ m/s}) \quad \Rightarrow \quad m_{water} = 5.0\times10^2\text{ kg}$$

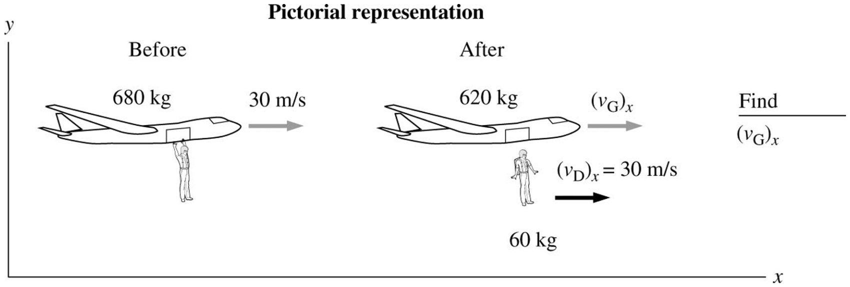
11.15. Model: Choose car + gravel to be the system. Ignore friction in the impulse approximation.
Visualize:



Solve: There are no *external* horizontal forces on the car + gravel system, so the horizontal momentum is conserved. This means $p_{fx} = p_{ix}$. Hence,

$$(10{,}000\text{ kg} + 4000\text{ kg})v_{fx} = (10{,}000\text{ kg})(2.0\text{ m/s}) + (4000\text{ kg})(0.0\text{ m/s}) \quad\Rightarrow\quad v_{fx} = 1.4\text{ m/s}$$

11.16. Model: Choose skydiver + glider to be the system in the impulse approximation. Ignore air resistance.
Visualize:



Note that there are no *external* forces in the *x*-direction (ignoring friction in the impulse approximation), implying conservation of momentum along the *x*-direction.
Solve: The momentum conservation equation $p_{fx} = p_{ix}$ gives

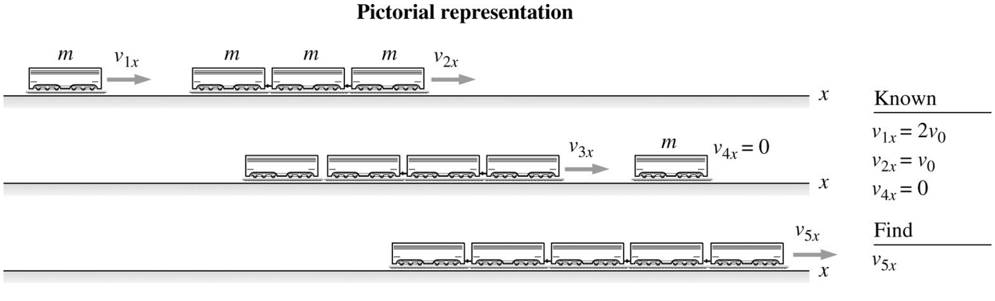
$$(680\text{ kg} - 60\text{ kg})(v_G)_x + (60\text{ kg})(v_D)_x = (680\text{ kg})(30\text{ m/s})$$

Immediately after release, the skydiver's horizontal velocity is still $(v_D)_x = 30$ m/s because he experiences no net horizontal force. Thus

$$(620\text{ kg})(v_G)_x + (60\text{ kg})(30\text{ m/s}) = (680\text{ kg})(30\text{ m/s}) \quad\Rightarrow\quad (v_G)_x = 30\text{ m/s}$$

Assess: The skydiver's motion in the vertical direction has *no* influence on the glider's horizontal motion. Notice that we did not need to invoke conservation of momentum to solve this problem. Because there are no external horizontal forces acting on either the skydiver or the glider, neither will change their horizontal speed when the skydiver lets go!

11.17. Model: Model the train cars as particles. Since the train cars stick together, we are dealing with perfectly inelastic collisions. Momentum is conserved in the collisions of this problem in the impulse approximation, in which we ignore external forces during the time of the collision.
Visualize:



Solve: In the collision between the three-car train and the single car:

$$mv_{1x} + (3m)v_{2x} = 4mv_{3x} \quad\Rightarrow\quad v_{1x} + 3v_{2x} = 4v_{3x} \quad\Rightarrow\quad v_{3x} = \tfrac{1}{4}v_{1x} + \tfrac{3}{4}v_{2x}$$

In the collision between the four-car train and the stationary car:

$$(4m)v_{3x} + m v_{4x} = (5m)v_{5x} \quad \Rightarrow \quad 4v_{3x} + 0 \text{ m/s} = 5v_{5x}$$

$$v_{5x} = \tfrac{4}{5}v_{3x} = \tfrac{1}{5}v_{1x} + \tfrac{3}{5}v_{2x} = v_0$$

11.31. Model: Assume that the momentum is conserved in the collision.
Solve: Applying conservation of momentum in the x- and y-directions yields

$$(p_{\text{f}x})_1 + (p_{\text{f}x})_2 = (p_{\text{i}x})_1 + (p_{\text{i}x})_2 \quad \Rightarrow \quad (p_{\text{f}x})_1 + 0 \text{ kg m/s} = -3 \text{ kg m/s} + 4 \text{ kg m/s} \quad \Rightarrow \quad (p_{\text{f}x})_1 = 1 \text{ kg}\cdot\text{m/s}$$

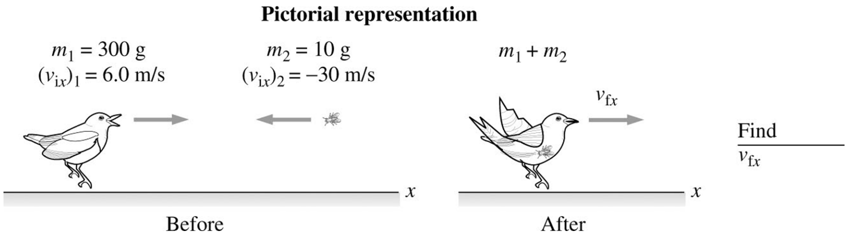
$$(p_{\text{f}y})_1 + (p_{\text{f}y})_2 = (p_{\text{i}y})_1 + (p_{\text{i}y})_2 \quad \Rightarrow \quad (p_{\text{f}y})_1 - 2 \text{ kg m/s} = 2 \text{ kg m/s} + 1 \text{ kg m/s} \quad \Rightarrow \quad (p_{\text{f}y})_1 = 5 \text{ kg}\cdot\text{m/s}$$

Thus, the final momentum of particle 1 is $(\vec{p}_\text{f})_1 = (1\hat{i} + 5\hat{j}) \text{ kg}\cdot\text{m/s}$

Assess: As a check, the vector sum of the initial momenta should equal the vector sum of the final momenta.

Section 11.3 Collisions

11.18. Model: We will define our system to be bird + bug. This is the case of an inelastic collision because the bird and bug move together after the collision. Horizontal momentum is conserved because there are no external forces acting on the system during the collision in the impulse approximation.
Visualize:

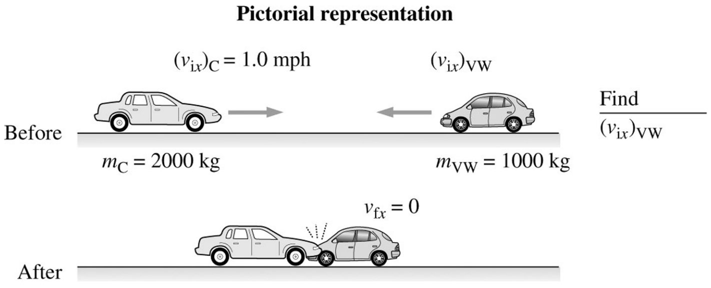


Solve: The conservation of momentum equation $p_{\text{f}x} = p_{\text{i}x}$ gives

$$(m_1 + m_2)v_{\text{f}x} = m_1(v_{\text{i}x})_1 + m_2(v_{\text{i}x})_2 \quad \Rightarrow \quad (300 \text{ g} + 10 \text{ g})v_{\text{f}x} = (300 \text{ g})(6.0 \text{ m/s}) + (10 \text{ g})(-30 \text{ m/s}) \quad \Rightarrow \quad v_{\text{f}x} = 4.8 \text{ m/s}$$

Assess: We left masses in grams, rather than convert to kilograms, because the mass units cancel out from both sides of the equation. Note that $(v_{\text{i}x})_2$ is negative because the bug is flying to the left.

11.19. Model: The two cars are not an isolated system because of external frictional forces. But during the collision friction is not going to be significant. Within the impulse approximation, the momentum of the Cadillac + Volkswagen system will be conserved in the collision.
Visualize:



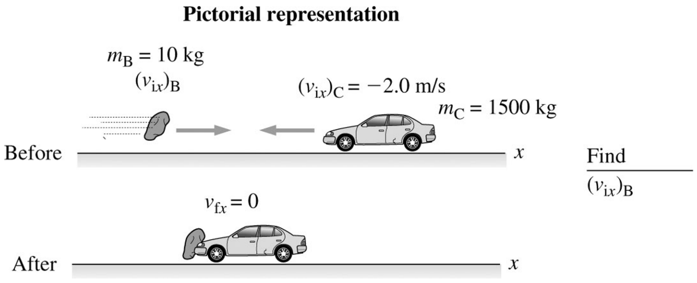
Solve: The momentum conservation equation $p_{\text{f}x} = p_{\text{i}x}$ gives

$$(m_\text{C} + m_\text{VW})v_{\text{f}x} = m_\text{C}(v_{\text{i}x})_\text{C} + m_\text{VW}(v_{\text{i}x})_\text{VW}$$

$$0 \text{ kg mph} = (2000 \text{ kg})(1.0 \text{ mph}) + (1000 \text{ kg})(v_{\text{i}x})_\text{VW} \quad\Rightarrow\quad (v_{\text{i}x})_\text{VW} = -2.0 \text{ mph}$$

so you need a *speed* of 2.0 mph.

11.20. Model: Because of external friction and drag forces, the car and the blob of sticky clay are not exactly an isolated system. But during the collision, friction and drag are not going to be significant. The momentum of the system will be conserved in the collision, within the impulse approximation.
Visualize:



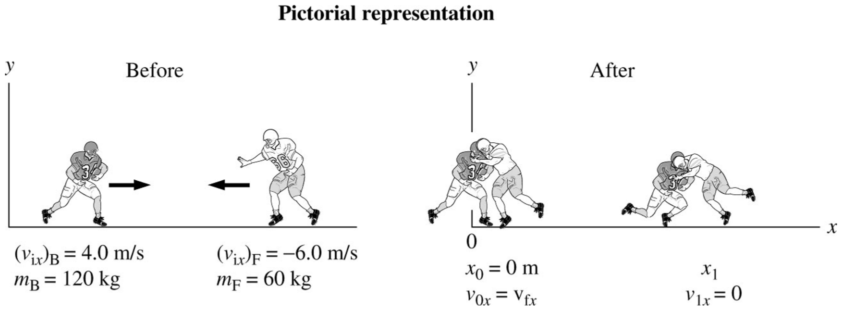
Solve: The conservation of momentum equation $p_{\text{f}x} = p_{\text{i}x}$ gives

$$(m_\text{C} + m_\text{B})(v_\text{f})_x = m_\text{B}(v_{\text{i}x})_\text{B} + m_\text{C}(v_{\text{i}x})_\text{C}$$

$$0 \text{ kg m/s} = (10 \text{ kg})(v_{\text{i}x})_\text{B} + (1500 \text{ kg})(-2.0 \text{ m/s}) \quad\Rightarrow\quad (v_{\text{i}x})_\text{B} = 3.0\times10^2 \text{ m/s}$$

Assess: This speed of the blob is around 600 mph, which is very large. However, a very large speed is *expected* in order to stop a car with only 10 kg of clay.

11.21. Model: This is a two-part problem. First, we have an inelastic collision between Fred (F) and Brutus (B). Fred and Brutus are an isolated system. The momentum of the system during collision is conserved since no significant external force acts on the system. The second part involves the dynamics of the Fred + Brutus system sliding on the ground.
Visualize:



Note that the collision is head-on and therefore one-dimensional.
Solve: The equation $p_{\text{f}x} = p_{\text{i}x}$ gives

$$(m_\text{F} + m_\text{B})v_{\text{f}x} = m_\text{F}(v_{\text{i}x})_\text{F} + m_\text{B}(v_{\text{i}x})_\text{B} \quad\Rightarrow\quad (60 \text{ kg} + 120 \text{ kg})v_{\text{f}x} = (60 \text{ kg})(-6.0 \text{ m/s}) + (120 \text{ kg})(4.0 \text{ m/s})$$
$$v_{\text{f}x} = 0.667 \text{ m/s}$$

The positive value indicates that the motion is in the direction of Brutus.

The model of kinetic friction yields:

$$f_k = -\mu_k n = -\mu_k (m_F + m_B)g = (m_F + m_B)a_x \quad \Rightarrow \quad a_x = -\mu_k g$$

Using the kinematic equation $v_{1x}^2 = v_{0x}^2 + 2a_x(x_1 - x_0)$, we get

$$v_{1x}^2 = v_{0x}^2 - 2\mu_k g\, x_1 \quad \Rightarrow \quad 0\text{ m}^2/\text{s}^2 = v_{fx}^2 - 2(0.30)(9.8\text{ m/s}^2)x_1$$

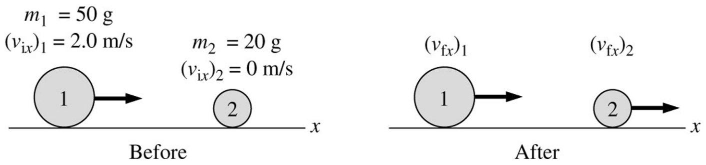
$$0\text{ m}^2/\text{s}^2 = (0.667\text{ m/s})^2 - (5.9\text{ m/s}^2)x_1 \quad \Rightarrow \quad x_1 = 7.6\text{ cm}$$

They slide 7.6 cm in the direction Brutus was running.

Assess: After the collision, Fred and Brutus slide with a small speed but with a good amount of kinetic friction. A stopping distance of 7.6 cm is reasonable.

11.22. Model: We assume this is a one-dimensional collision that obeys the conservation laws of momentum and mechanical energy.

Visualize:



Note that momentum conservation alone is not sufficient to solve this problem because the two final velocities $(v_{fx})_1$ and $(v_{fx})_2$ are unknowns and cannot be determined from one equation.

Solve: Momentum conservation: $m_1(v_{ix})_1 + m_2(v_{ix})_2 = m_1(v_{fx})_1 + m_2(v_{fx})_2$

Energy conservation: $\frac{1}{2}m_1(v_{ix})_1^2 + \frac{1}{2}m_2(v_{ix})_2^2 = \frac{1}{2}m_1(v_{fx})_1^2 + \frac{1}{2}m_2(v_{fx})_2^2$

These two equations can be solved for $(v_{fx})_1$ and $(v_{fx})_2$, as shown by Equations 11.26 through 11.30, to give

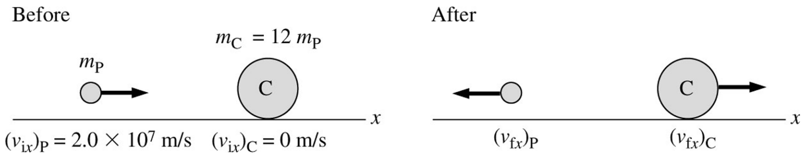
$$(v_{fx})_1 = \frac{m_1 - m_2}{m_1 + m_2}(v_{ix})_1 = \frac{50\text{ g} - 20\text{ g}}{50\text{ g} + 20\text{ g}}(2.0\text{ m/s}) = 0.86\text{ m/s}$$

$$(v_{fx})_2 = \frac{2m_1}{m_1 + m_2}(v_{ix})_1 = \frac{2(50\text{ g})}{50\text{ g} + 20\text{ g}}(2.0\text{ m/s}) = 2.9\text{ m/s}$$

Assess: These velocities are of a reasonable magnitude. Since both these velocities are positive, both balls move along the +*x*-direction.

11.23. Model: This is a case of a perfectly elastic collision between a proton and a carbon atom. The collision obeys the momentum as well as the energy conservation law.

Visualize:



Solve: Momentum conservation: $m_P(v_{ix})_P + m_C(v_{ix})_C = m_P(v_{fx})_P + m_C(v_{fx})_C$

Energy conservation: $\frac{1}{2}m_P(v_{ix})_P^2 + \frac{1}{2}m_C(v_{ix})_C^2 = \frac{1}{2}m_P(v_{fx})_P^2 + \frac{1}{2}m_C(v_{fx})_C^2$

These two equations can be solved, as described in the text through Equations 11.25 through 11.29:

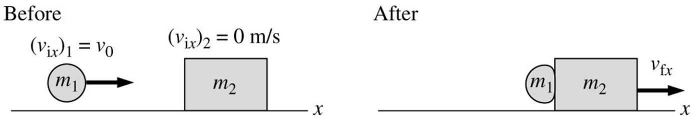
$$(v_{fx})_P = \frac{m_P - m_C}{m_P + m_C} m_P + m_C (v_{ix})_P = \left(\frac{m_P - 12m_P}{m_P + 12m_P}\right)(2.0\times10^7\ \text{m/s}) = -1.69\times10^7\ \text{m/s}$$

$$(v_{fx})_C = \frac{2m_P}{m_P + m_C}(v_{ix})_P = \left(\frac{2m_P}{m_P + 12m_P}\right)(2.0\times10^7\ \text{m/s}) = 3.1\times10^6\ \text{m/s}$$

After the elastic collision the proton rebounds at 1.69×10^7 m/s and the carbon atom moves forward at 3.08×10^6 m/s.

11.24. Model: This is the case of a perfectly inelastic collision. Momentum is conserved because no external force acts on the system $(\text{clay} + \text{brick})$. We also represent our system as a particle.

Visualize:



Solve: (a) The conservation of momentum equation $p_{fx} = p_{ix}$ is

$$(m_1 + m_2)v_{fx} = m_1(v_{ix})_1 + m_2(v_{ix})_2$$

Using $(v_{ix})_1 = v_0$ and $(v_{ix})_2 = 0$, we get

$$v_{fx} = \frac{m_1}{m_1 + m_2}(v_{ix})_1 = \frac{0.050\ \text{kg}}{(1.0\ \text{kg} + 0.050\ \text{kg})}(v_{ix})_1 = 0.0476(v_{ix})_1 = 0.0476\, v_0$$

The brick is moving with speed $0.048v_0$.

(b) The initial and final kinetic energies are given by

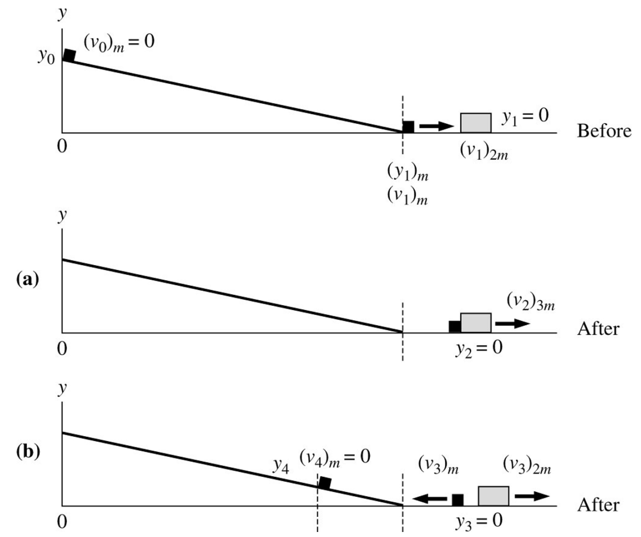
$$K_i = \frac{1}{2}m_1(v_{ix})_1^2 + \frac{1}{2}m_2(v_{ix})_2^2 = \frac{1}{2}(0.050\ \text{kg})v_0^2 + \frac{1}{2}(1.0\ \text{kg})(0\ \text{m/s})^2 = (0.025\ \text{kg})v_0^2$$

$$K_f = \frac{1}{2}(m_1 + m_2)v_{fx}^2 = \frac{1}{2}(1.0\ \text{kg} + 0.050\ \text{kg})(0.0476)^2 v_0^2 = 0.00119\, v_0^2$$

$$\text{The percent of energy lost} = \left(\frac{K_i - K_f}{K_i}\right)\times100\% = \left(1 - \frac{0.00119}{0.025}\right)\times100\% = 95\%$$

11.25. Model: Model the two packages as particles. Momentum is conserved in both inelastic and elastic collisions. Kinetic energy is conserved only in a perfectly elastic collision.

Visualize:



Solve: For a package with mass *m* the conservation of energy equation is

$$K_1 + U_{g1} = K_0 + U_{g0} \Rightarrow \frac{1}{2}m(v_1)_m^2 + mgy_1 = \frac{1}{2}m(v_0)_m^2 + mgy_0$$

Using $(v_0)_m = 0$ m/s and $y_1 = 0$ m,

$$\frac{1}{2}m(v_1)_m^2 = mgy_0 \Rightarrow (v_1)_m = \sqrt{2gy_0} = \sqrt{2(9.8\text{ m/s}^2)(3.0\text{ m})} = 7.668\text{ m/s}$$

(a) For the perfectly inelastic collision the conservation of momentum equation is

$$p_{fx} = p_{ix} \Rightarrow (m+2m)(v_2)_{3m} = m(v_1)_m + (2m)(v_1)_{2m}$$

Using $(v_1)_{2m} = 0$ m/s, we get

$$(v_2)_{3m} = (v_1)_m/3 = 2.56\text{ m/s}$$

The packages move off together at a speed of 2.6 m/s.
(b) For the elastic collision, the mass *m* package rebounds with velocity

$$(v_3)_m = \frac{m-2m}{m+2m}(v_1)_m = -\frac{1}{3}(7.668\text{ m/s}) = -2.56\text{ m/s}$$

The negative sign with $(v_3)_m$ shows that the package with mass *m* rebounds and goes to the position y_4. We can determine y_4 by applying the conservation of energy equation as follows. For a package of mass *m*:

$$K_f + U_{gf} = K_i + U_{gi} \Rightarrow \frac{1}{2}m(v_4)_m^2 + mgy_4 = \frac{1}{2}m(v_3)_m^2 + mgy_3$$

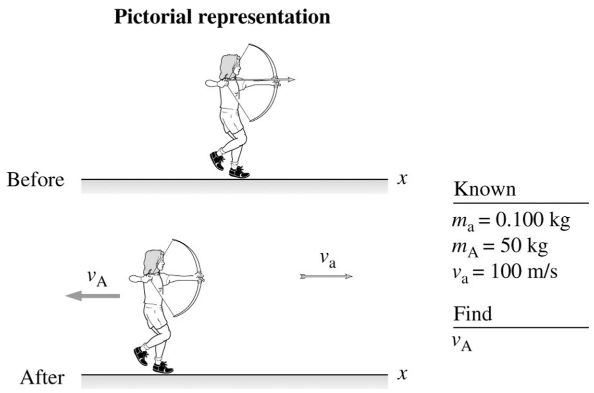
Using $(v_3)_m = -2.55$ m/s, $y_3 = 0$ m, and $(v_4)_m = 0$ m/s, we get

$$mgy_4 = \frac{1}{2}m(-2.56\text{ m/s})^2 \Rightarrow y_4 = 33\text{ cm}$$

Section 11.4 Explosions

11.26. Model: We will define our system to be archer + arrow. The force of the archer (A) on the arrow **(a)** is equal to the force of the arrow on the archer. These are internal forces within the system. The archer is standing on frictionless ice, and the normal force by ice on the system balances the weight force. Thus $\vec{F}_{\text{ext}} = \vec{0}$ on the system, and momentum is conserved.

Visualize:



The initial momentum p_{ix} of the system is zero, because the archer and the arrow are at rest. The final moment p_{fx} must also be zero.

Solve: We have $M_A v_A + m_a v_a = 0$ kg m/s. Therefore,

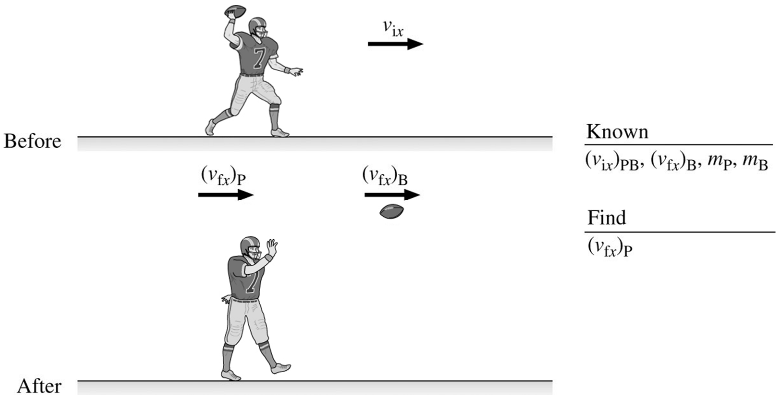
$$v_A = \frac{-m_a v_a}{m_A} = \frac{-(0.100\text{ kg})(100\text{ m/s})}{50\text{ kg}} = -0.20\text{ m/s}$$

The archer's recoil *speed* is 0.20 m/s.

Assess: It is the total final momentum that is zero, although the individual momenta are nonzero. Since the arrow has forward momentum, the archer will have backward momentum.

11.27. Model: We will define our system to be the football player (P) and the football (B). Their interaction is an explosion because the force involved is internal to the P + B system. There are no external horizontal forces present on either of the two, so horizontal momentum is conserved.

Visualize:



The system has nonzero initial momentum p_{ix}, which must be conserved.

Solve: (a) The final velocity of the ball is $(v_{fx})_B = 15.0$ m/s. Equating the initial and final momentum gives $m_P(v_{fx})_P + m_B(v_{fx})_B = (m_B + m_P)v_{ix}$. Solving for $(v_{fx})_P$ gives

$$(v_{fx})_P = \frac{(m_B + m_P)v_{ix} - m_B(v_{fx})_B}{m_P} = \frac{(0.450\text{ kg} + 70.0\text{ kg})(2.00\text{ m/s}) - (0.450\text{ kg})(15.0\text{ m/s})}{70.0\text{ kg}} = 1.92\text{ m/s}$$

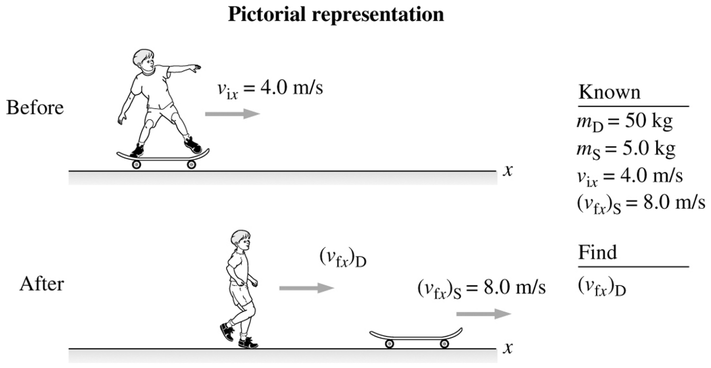
(b) The final velocity of the ball is $(v_{fx})_B = (v_{fx})_P + 15.0\text{ m/s}$. Inserting this into the equation for conservation of momentum and solving for $(v_{fx})_B$ gives

$$m_P(v_{fx})_P + m_B\left[(v_{fx})_P + 15.0\text{ m/s}\right] = (m_B + m_P)v_{ix}.$$

$$(v_{fx})_P = \frac{(m_B + m_P)v_{ix} - m_B(15.0\text{ m/s})}{m_P + m_B} = \frac{(0.450\text{ kg} + 70.0\text{ kg})(2.00\text{ m/s}) - (0.450\text{ kg})(15.0\text{ m/s})}{70.0\text{ kg} + 0.450\text{ kg}} = 1.90\text{ m/s}$$

Assess: In part **(b)**, the final velocity of the ball is greater than in part **(a)**, so the player's final velocity is slightly less so that momentum is conserved.

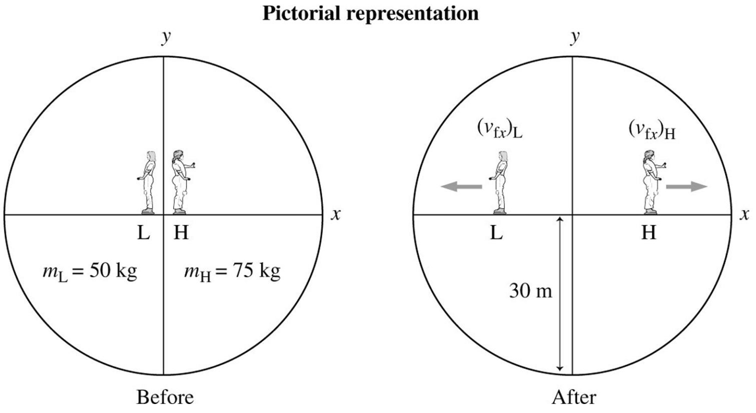
11.28. Model: We will define our system to be Dan + skateboard, and their interaction as an explosion. While friction is present between the skateboard and the ground, it is negligible in the impulse approximation.
Visualize:


Pictorial representation

The system has nonzero initial momentum p_{ix}. As Dan (D) jumps backward off the gliding skateboard (S), the skateboard will move forward so that the final total momentum of the system p_{fx} is equal to p_{ix}.
Solve: We have $m_S(v_{fx})_S + m_D(v_{fx})_D = (m_S + m_D)v_{ix}$. Thus,

$$(5.0\text{ kg})(8.0\text{ m/s}) + (50\text{ kg})(v_{fx})_D = (5.0\text{ kg} + 50\text{ kg})(4.0\text{ m/s}) \;\Rightarrow\; (v_{fx})_D = 3.6\text{ m/s}$$

11.29. Model: This problem deals with a case that is the opposite of a collision. The two ice skaters, heavier and lighter, will be modeled as particles. The skaters (or particles) move apart after pushing off against each other. During the "explosion," the total momentum of the system is conserved.
Visualize:


Pictorial representation

Solve: The initial momentum is zero. Thus the conservation of momentum equation $p_{fx} = p_{ix}$ gives

$$m_H(v_{fx})_H + m_L(v_{fx})_L = 0 \text{ kg m/s} \quad \Rightarrow \quad (75 \text{ kg})(v_{fx})_H + (50 \text{ kg})(v_{fx})_L = 0 \text{ kg m/s}$$

Using the observation that the heavier skater takes 20 s to cover a distance of 30 m, we find $(v_{fx})_H = (30 \text{ m})/(20 \text{ s}) = 1.5 \text{ m/s}$. Thus,

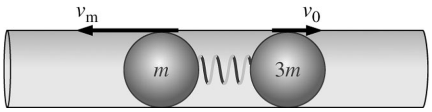
$$(75 \text{ kg})(1.5 \text{ m/s}) + (50 \text{ kg})(v_{fx})_L = 0 \text{ kg m/s} \quad \Rightarrow \quad (v_{fx})_L = -2.25 \text{ m/s}$$

Thus, the time for the lighter skater to reach the edge is

$$\frac{30 \text{ m}}{(v_{fx})_L} = \frac{30 \text{ m}}{2.25 \text{ m/s}} = 13 \text{ s}$$

Assess: Conservation of momentum leads to a higher speed for the lighter skater, and hence a shorter time to reach the edge of the ice rink.

11.30. Model: Assume that the tube is frictionless and ignore air resistance. Model the two balls as particles.
Visualize:



Solve: The initial momentum of the system is zero because both balls are stationary. Therefore, conservation of momentum tells us that the final momentum of the system must be zero:

$$mv_m + 3mv_0 = 0 \quad \Rightarrow \quad v_m = -3v_0$$

Thus, the speed of the lighter ball is $3v_0$.

Assess: The negative sign indicates that the lighter ball moves in the direction opposite the larger ball.

Section 11.5 Momentum in Two Dimensions

11.31. Model: Assume that the momentum is conserved in the collision.
Visualize: Please refer to Figure EX11.31.
Solve: Applying conservation of momentum in the x- and y-directions yields

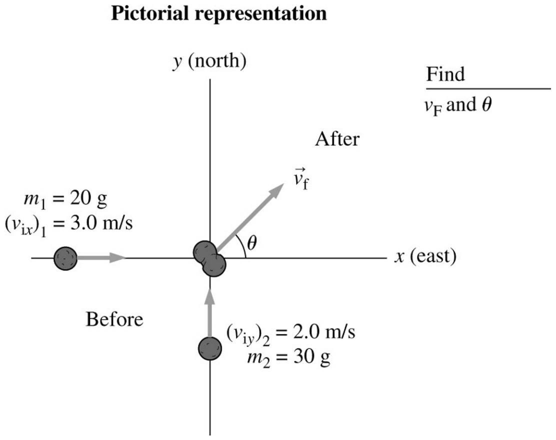
$$(p_{fx})_1 + (p_{fx})_2 = (p_{ix})_1 + (p_{ix})_2 \quad \Rightarrow \quad (p_{fx})_1 + 0 \text{ kg m/s} = -2 \text{ kg m/s} + 4 \text{ kg m/s} \quad \Rightarrow \quad (p_{fx})_1 = 2 \text{ kg m/s}$$
$$(p_{fy})_1 + (p_{fy})_2 = (p_{iy})_1 + (p_{iy})_2 \quad \Rightarrow \quad (p_{fy})_1 - 1 \text{ kg m/s} = 2 \text{ kg m/s} + 1 \text{ kg m/s} \quad \Rightarrow \quad (p_{fy})_1 = 4 \text{ kg m/s}$$

Thus, the final momentum of particle 1 is $(\vec{p}_f)_1 = (2\hat{i} + 4\hat{j})$ kg m/s.

11.32. Since the object is initially at rest, its total momentum is zero. After it explodes the total momentum of the fragments must also be zero. With $\vec{p}_1 = (-2, 2)$ kg m/s and $\vec{p}_2 = (3, 0)$ kg m/s, the requirement that $\vec{p}_1 + \vec{p}_2 + \vec{p}_3 = 0$ means that $\vec{p}_3 = (-1, -2)$ kg m/s.

11.33. Model: This problem deals with the conservation of momentum in two dimensions in an inelastic collision.

Visualize:



Solve: The conservation of momentum equation $\vec{p}_{\text{before}} = \vec{p}_{\text{after}}$ gives

$$m_1(v_{ix})_1 + m_2(v_{ix})_2 = (m_1 + m_2)v_{fx},\ m_1(v_{iy})_1 + m_2(v_{iy})_2 = (m_1 + m_2)v_{fy}$$

Substituting in the given values, we find

$$(0.020\text{ kg})(3.0\text{ m/s}) + 0.0\text{ kg m/s} = (0.020\text{ kg} + 0.030\text{ kg})v_f\cos\theta$$

$$0.0\text{ kg m/s} + (0.030\text{ kg})(2.0\text{ m/s}) = (0.020\text{ kg} + 0.030\text{ kg})v_f\sin\theta$$

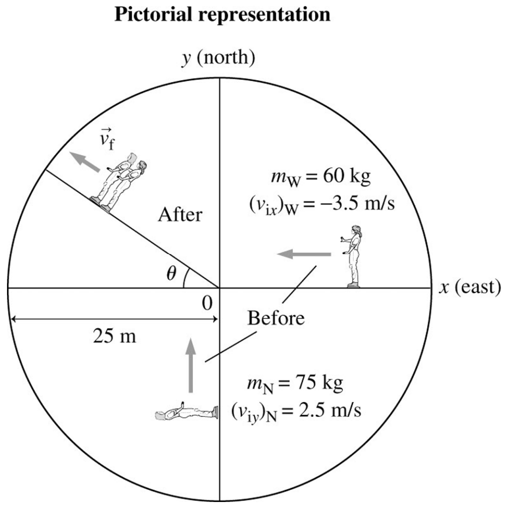
$$v_f\cos\theta = 1.2\text{ m/s},\ v_f\sin\theta = 1.2\text{ m/s}$$

$$v_f = \sqrt{(1.2\text{ m/s})^2 + (1.2\text{ m/s})^2} = 1.7\text{ m/s},\ \theta = \tan^{-1}\frac{v_y}{v_x} = \tan^{-1}(1) = 45°$$

The ball of clay moves 45° north of east at 1.7 m/s.

11.34. Model: Model the skaters as particles. The two skaters, one traveling north (N) and the other traveling west (W), are the system. Since the two skaters hold together after the "collision," this is a case of a perfectly inelastic collision in two dimensions. Momentum is conserved since no significant external force in the *x*-*y* plane acts on the system during the "collision."

Visualize:



Solve: **(a)** Applying conservation of momentum in the *x*-direction gives

$$(m_\text{N}+m_\text{W})v_{\text{f}x}=m_\text{N}(v_{\text{i}x})_\text{N}+m_\text{W}(v_{\text{i}x})_\text{W} \quad\Rightarrow\quad (75\text{ kg}+60\text{ kg})v_{\text{f}x}=0\text{ kg m/s}+(60\text{ kg})(-3.5\text{ m/s})$$
$$v_{\text{f}x}=-1.556\text{ m/s}$$

Applying conservation of momentum in the *y*-direction gives

$$(m_\text{N}+m_\text{W})v_{\text{f}y}=m_\text{N}(v_{\text{i}y})_\text{N}+m_\text{W}(v_{\text{i}y})_\text{W} \quad\Rightarrow\quad (75\text{ kg}+60\text{ kg})v_{\text{f}y}=(75\text{ kg})(2.5\text{ m/s})+0\text{ kg m/s}$$
$$v_{\text{f}y}=1.389\text{ m/s}$$

The final speed is therefore

$$v_\text{f}=\sqrt{(v_{\text{f}x})^2+(v_{\text{f}y})^2}=2.085\text{ m/s}$$

The time to glide to the edge of the rink is

$$\frac{\text{radius of the rink}}{v_\text{f}}=\frac{25\text{ m}}{2.085\text{ m/s}}=12\text{ s}$$

(b) The location is $\theta=\tan^{-1}(v_{\text{f}y}/v_{\text{f}x})=42^\circ$ north of west.

Assess: A time of 12 s in covering a distance of 25 m at a speed of ≈ 2 m/s is reasonable.

11.35. Model: Ignore gravitational forces.

Visualize: We are given the rate of fuel burn $R=|dm/dt|=(250\text{ kg})/(30\text{ s})=8.33\text{ kg/s}$.

Solve: The formula relating the rocket thrust with the exhaust speed is

$$F_\text{thrust}=v_\text{ex}R\Rightarrow v_\text{ex}=\frac{F_\text{thrust}}{R}=\frac{15\text{ kN}}{8.33\text{ kg/s}}=1800\text{ m/s}$$

Assess: We expect a rocket exhaust speed to be high.

Section 11.6 Rocket Propulsion

11.36. Model: Ignore gravitational forces.

Visualize: We are given $m_\text{R}=150$ kg, $v_\text{ex}=2500$ m/s, $v_\text{max}=4000$ m/s.

Solve: Solve the formula relating these variables for m_F0.

$$v_\text{max}=v_\text{ex}\ln\left(\frac{m_\text{R}+m_\text{F0}}{m_\text{R}}\right)\Rightarrow m_\text{F0}=m_\text{R}\left(e^{v_\text{max}/v_\text{ex}}-1\right)=(150\text{ kg})\left(e^{4000/2500}-1\right)=590\text{ kg}$$

Assess: The initial amount of fuel is a few times the mass of the empty rocket.

11.37. Model: Ignore gravitational forces.

Visualize: We are given $v_\text{ex}=2000$ m/s. We are also given $m_\text{F0}=5m_\text{R}$.

Solve: We need the initial and final masses.

$$m_0=m_\text{R}+m_\text{F0}=m_\text{R}+5m_\text{R}=6m_\text{R}$$
$$m=m_\text{R}+m_\text{F}=m_\text{R}+\tfrac{1}{2}m_\text{F0}=\tfrac{2}{2}m_\text{R}+\tfrac{1}{2}(5m_\text{R})=\tfrac{7}{2}m_\text{R}$$

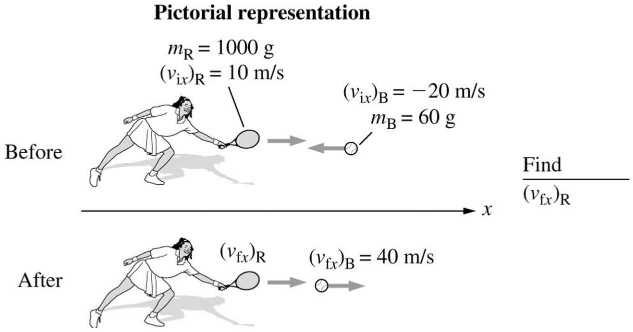
Now we put it all together.

$$v=v_\text{ex}\ln\left(\frac{m_0}{m}\right)=v_\text{ex}\ln\left(\frac{6m_\text{R}}{\frac{7}{2}m_\text{R}}\right)=(2000\text{ m/s})\ln\left(\frac{12}{7}\right)=1000\text{ m/s}$$

Assess: The rocket is only going half the exhaust speed of the fuel, but only half of the fuel has been used so far; the rocket can still gain a lot of speed.

Problems

11.38. Model: Let the system be ball + racket. During the collision of the ball and racket, momentum is conserved because all external interactions are insignificantly small.
Visualize:



Solve: (a) The conservation of momentum equation $p_{fx} = p_{ix}$ gives

$$m_R(v_{fx})_R + m_B(v_{fx})_B = m_R(v_{ix})_R + m_B(v_{ix})_B$$

$$(1.000 \text{ kg})(v_{fx})_R + (0.060 \text{ kg})(40 \text{ m/s}) = (1.000 \text{ kg})(10 \text{ m/s}) + (0.060 \text{ kg})(-20 \text{ m/s}) \quad \Rightarrow \quad (v_{fx})_R = 6.4 \text{ m/s}$$

(b) The impulse on the ball is calculated from $(p_{fx})_B = (p_{ix})_B + J_x$ as follows:

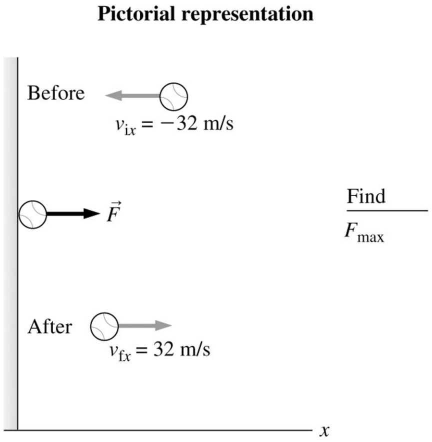
$$(0.060 \text{ kg})(40 \text{ m/s}) = (0.060 \text{ kg})(-20 \text{ m/s}) + J_x \quad \Rightarrow \quad J_x = 3.6 \text{ N s} = \int F dt = F_{\text{avg}} \Delta t$$

$$F_{\text{avg}} = \frac{3.6 \text{ Ns}}{10 \text{ ms}} = 3.6 \times 10^2 \text{ N}$$

Let us now compare this force with the gravitational force on the ball $(F_G)_B = m_B g = (0.060 \text{ kg})(9.8 \text{ m/s}^2) = 0.588$ N. We find $F_{\text{avg}} = 612(F_G)_B$.

Assess: This is a significant force and is reasonable because the impulse due to this force not only changes the direction of the ball also but changes the speed of the ball from approximately 45 mph to 90 mph.

11.39. Model: Model the tennis ball as a particle, and its interaction with the wall as a collision.
Visualize:



The force increases to $F_{\max}$ during the first two ms, stays at $F_{\max}$ for two ms, and then decreases to zero during the last two ms. The graph shows that F_x is positive, so the force acts to the right.

Solve: Using the impulse-momentum theorem $p_{\text{f}x} = p_{\text{i}x} + J_x$, we find

$$(0.06\text{ kg})(32\text{ m/s}) = (0.06\text{ kg})(-32\text{ m/s}) + \int_0^{6\text{ ms}} F_x(t)dt$$

The impulse is

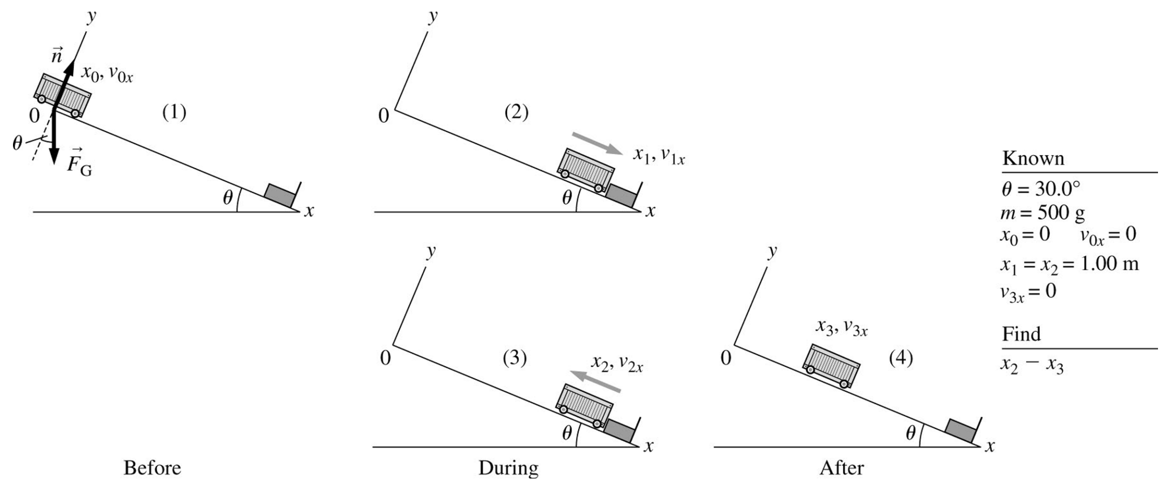
$$J_x = \int_0^{6\text{ ms}} F_x(t)dt = \text{area under force curve} = \frac{1}{2}F_{\text{max}}(0.0020\text{ s}) + F_{\text{max}}(0.0020\text{ s}) + \frac{1}{2}F_{\text{max}}(0.0020\text{ s}) = (0.0040\text{ s})F_{\text{max}}$$

$$F_{\text{max}} = \frac{(0.060\text{ kg})(32\text{ m/s}) + (0.060\text{ kg})(32\text{ m/s})}{0.0040\text{ s}} = 9.6\times10^2\text{ N}$$

11.40. Model: Model the cart as a particle sliding down a frictionless ramp. The cart is subjected to an impulsive force when it comes in contact with a rubber block at the bottom of the ramp. We shall use the impulse-momentum theorem and the constant-acceleration kinematic equations.

Visualize:

Pictorial representation



Solve: From the free-body diagram on the cart, Newton's second law applied to the system before the collision gives

$$\Sigma(F)_x = F_G\sin\theta = ma_x \quad\Rightarrow\quad a_x = \frac{mg\sin\theta}{m} = g\sin30.0° = \frac{9.81\text{ m/s}^2}{2} = 4.905\text{ m/s}^2$$

Using this acceleration, we can find the cart's speed just before its contact with the rubber block:

$$v_{1x}^2 = v_{0x}^2 + 2a_x(x_1 - x_0) = 0\text{ m}^2/\text{s}^2 + 2(4.905\text{ m/s}^2)(1.00\text{ m} - 0\text{ m}) \quad\Rightarrow\quad v_{1x} = 3.132\text{ m/s}$$

Now we can use the impulse-momentum theorem to obtain the velocity just after the collision:

$$mv_{2x} = mv_{1x} + \int F_x dt = mv_{1x} + \text{area under the force graph}$$

$$(0.500\text{ kg})v_{2x} = (0.500\text{ kg})(3.13\text{ m/s}) - \tfrac{1}{2}(200\text{ N})(26.7\times10^{-3}\text{ s}) \quad\Rightarrow\quad v_{2x} = -2.208\text{ m/s}$$

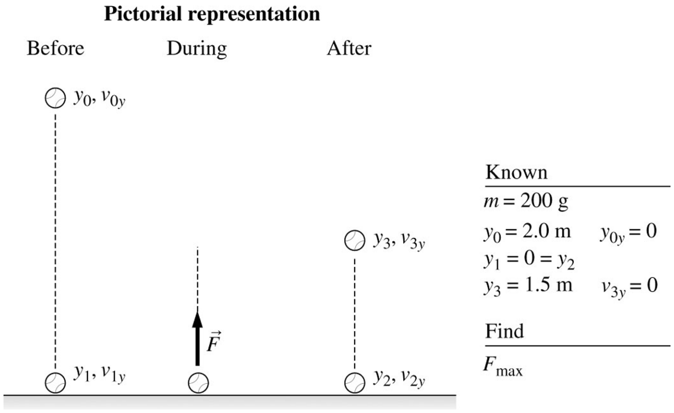
Note that the given force graph is positive, but in this coordinate system the impulse of the force is to the left (i.e., up the slope). That is the reason to put a minus sign while evaluating the $\int F_x dt$ integral.

We can once again use a kinematic equation to find how far the cart will roll back up the ramp:

$$v_{3x}^2 = v_{2x}^2 + 2a_x(x_3 - x_2) \quad\Rightarrow\quad (0\text{ m/s})^2 = (-2.208\text{ m/s})^2 + 2(-4.905\text{ m/s}^2)(x_3 - x_2) \quad\Rightarrow\quad (x_3 - x_2) = 0.497\text{ m}$$

11.41. Model: Model the ball as a particle that is subjected to an impulse when it is in contact with the floor. We shall also use constant-acceleration kinematic equations. During the collision, ignore any forces other than the interaction between the floor and the ball in the impulse approximation.

Visualize:



Solve: To find the ball's velocity just before and after it hits the floor:

$$v_{1y}^2 = v_{0y}^2 + 2a_y(y_1 - y_0) = 0\text{ m}^2/\text{s}^2 + 2(-9.8\text{ m/s}^2)(0 - 2.0\text{ m}) \quad\Rightarrow\quad v_{1y} = -6.261\text{ m/s}$$

$$v_{3y}^2 = v_{2y}^2 + 2a_y(y_3 - y_2) \quad\Rightarrow\quad 0\text{ m}^2/\text{s}^2 = v_{2y}^2 + 2(-9.8\text{ m/s}^2)(1.5\text{ m} - 0\text{ m}) \quad\Rightarrow\quad v_{2y} = 5.422\text{ m/s}$$

The force exerted by the floor on the ball can be found from the impulse-momentum theorem:

$$mv_{2y} = mv_{1y} + \int F dt = mv_{1y} + \text{area under the force curve}$$

$$(0.200\text{ kg})(5.422\text{ m/s}) = -(0.200\text{ kg})(6.261\text{ m/s}) + \tfrac{1}{2}F_{\max}(5.0\times10^{-3}\text{ s})$$

$$F_{\max} = 9.3\times10^2\text{ N}$$

Assess: A maximum force of 9.3×10^2 N exerted by the floor is reasonable. This force is the same order of magnitude as the force of the racket on the tennis ball in the Problem 11.38.

11.42. Solve: Apply Equation 11.7 and Newton's second law. The latter tells us that the average force used to expel the grains is

$$F_{\text{avg}} = ma = (1.0\times10^{-10}\text{ kg})(2.5\times10^4\text{ m/s}^2) = 2.5\times10^{-6}\text{ N}$$

Inserting this into Equation 11.7 gives

$$J = F_{\text{avg}}\Delta t = (2.5\times10^{-6}\text{ N})(3.0\times10^4\text{ s}) = 7.5\times10^{-10}\text{ kg m/s}$$

11.43. Solve: Using Newton's second law for the *x*-direction, $F_x = dp_x/dt$. Therefore,

$$F_x = \frac{d}{dt}(6t^2\text{ kg m/s}) = 12t\text{ N}$$

Assess: The *x*-component of the net force on an object is equal to the time rate of change of the *x*-component of the object's momentum.

11.44. Model: Model the gliders as particles and apply conservation of momentum. The 200 g glider will be labeled 1, the 300 g glider will be labeled 2, and the 400 g glider will be labeled 3.

Visualize: See Figure EX11.44.

Solve: To apply conservation of momentum, we need to calculate the speed of two gliders from the figure. Differentiating the position-versus-time curves gives:

$$v_{1x} = \frac{dy}{dt} = 0.53 \text{ m/s}, \; v_{2x} = \frac{dy}{dt} = -0.41 \text{ m/s}$$

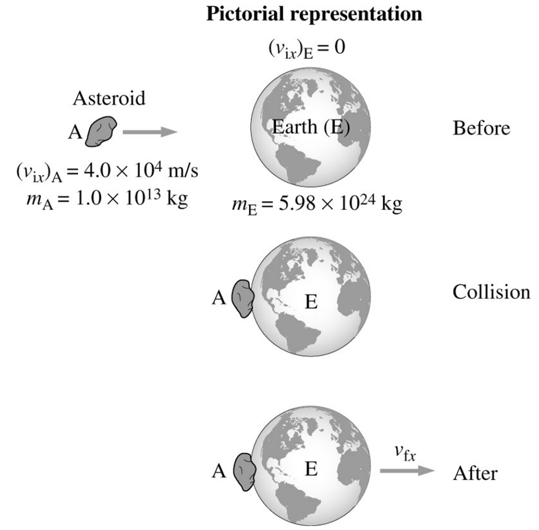
The initial momentum is zero because the three gliders are stationary. Therefore, conservation of momentum gives

$$m_1 v_{1x} + m_2 v_{2x} + m_3 v_{3x} = 0$$

$$v_{3x} = \tfrac{-1}{m_3}(m_1 v_{1x} + m_2 v_{2x}) = \tfrac{-1}{0.40 \text{ kg}}[(0.20 \text{ kg})(0.53 \text{ m/s}) + (0.30 \text{ kg})(-0.41 \text{ m/s})] = 0.043 \text{ m/s}$$

Thus, the 400 g glider flies of the right (because $v_{3x} > 0$) at a speed of 0.043 m/s.

11.45. Model: Model the earth (E) and the asteroid (A) as particles. Earth + asteroid is our system. Since the two stick together during the collision, this is a case of a perfectly inelastic collision. Momentum is conserved in the collision since no significant external force acts on the system.
Visualize:


Pictorial representation
$(v_{ix})_E = 0$
Asteroid
A
Earth (E)
Before
$(v_{ix})_A = 4.0 \times 10^4$ m/s
$m_A = 1.0 \times 10^{13}$ kg
$m_E = 5.98 \times 10^{24}$ kg
A
E
Collision
v_{fx}
A
E
After

Solve: **(a)** The conservation of momentum equation $p_{fx} = p_{ix}$ gives

$$m_A (v_{ix})_A + m_E (v_{ix})_E = (m_A + m_E) v_{fx}$$

$$(1.0\times10^{13} \text{ kg})(4.0\times10^4 \text{ m/s}) + 0 \text{ kg m/s} = (1.0\times10^{13} \text{ kg} + 5.98\times10^{24} \text{ kg}) v_{fx} \quad \Rightarrow \quad v_{fx} = 6.7\times10^{-8} \text{ m/s}$$

(b) The speed of the earth going around the sun is

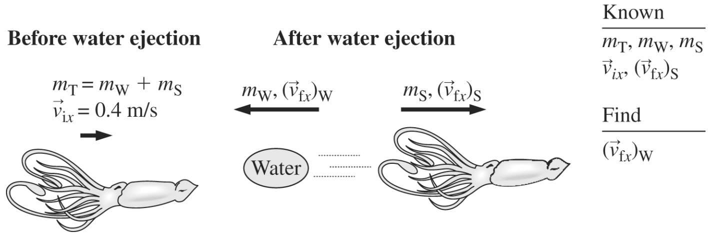
$$v_E = \frac{2\pi r}{T} = \frac{2\pi(1.50\times10^{11} \text{ m})}{3.15\times10^7 \text{ s}} = 3.0\times10^4 \text{ m/s}$$

Thus, $v_{fx}/v_E = 2.2\times10^{-12} = 2.2\times10^{-10}$.

Assess: The earth's recoil speed is insignificant compared to its orbital speed because of its large mass.

11.46. Model: Model the squid and the water ejected as particles and ignore drag forces during the short time interval over which the water is expelled (the impulse approximation). Because the external forces are negligible, momentum will be conserved.

Visualize:



Solve: Applying conservation of momentum gives

$$m_T v_{1x} = m_W (v_{fx})_W + m_S (v_{fx})_S$$

$$(v_{fx})_W = \frac{1}{m_W}[m_T v_{1x} - m_S (v_{fx})_S] = \frac{1}{0.10\text{ kg}}[(1.6\text{ kg})(0.4\text{ m/s}) - (1.5\text{ kg})(2.5\text{ m/s})] = -31.1\text{ m/s}$$

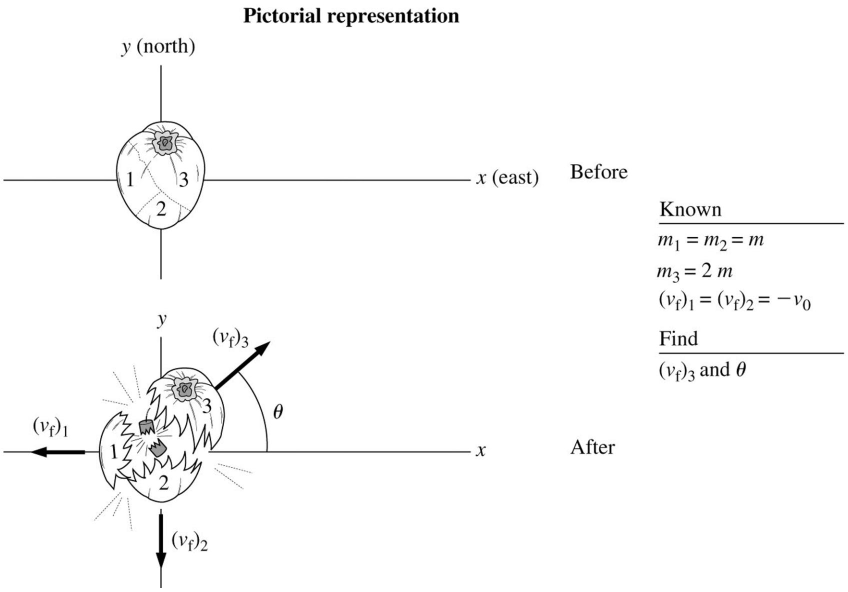
This water is ejected in the direction opposite the squid's initial velocity, so the speed with which the water is ejected relative to the squid is

$$v_{rel} = (v_{fx})_W - (v_{fx})_S = -31.1\text{ m/s} - 0.4\text{ m/s} = -31.5\text{ m/s}$$

or 32 m/s to two significant figures.

11.47. Model: This problem deals with a case that is the opposite of a collision. Our system comprises of three coconut pieces that are modeled as particles. During the explosion, the total momentum of the system is conserved in the *x*- and *y*-directions.

Visualize:



Solve: The initial momentum is zero. From $p_{fx} = p_{ix}$, we get

$$+m_1(v_f)_1 + m_3(v_f)_3\cos\theta = 0\text{ kg m/s} \quad\Rightarrow\quad (v_f)_3\cos\theta = \frac{-m_1(v_f)_1}{m_3} = \frac{-m(-v_0)}{2m} = \frac{v_0}{2}$$

From $p_{\text{fx}} = p_{\text{ix}}$, we get

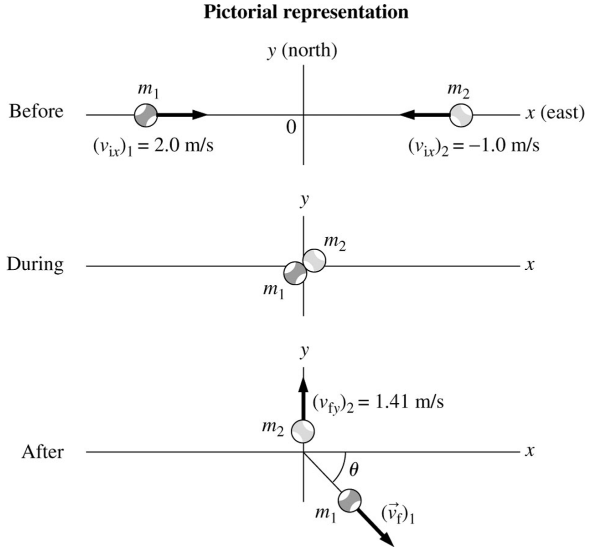
$$+m_2(v_\text{f})_2 + m_3(v_\text{f})_3\sin\theta = 0 \text{ kg m/s} \quad \Rightarrow \quad (v_\text{f})_3\sin\theta = \frac{-m_2(v_\text{f})_2}{m_3} = \frac{-m(-v_0)}{2m} = \frac{v_0}{2}$$

$$(v_\text{f})_3 = \sqrt{\left(\frac{v_0}{2}\right)^2 + \left(\frac{v_0}{2}\right)^2} = \frac{v_0}{\sqrt{2}}, \; \theta = \tan^{-1}(1) = 45°$$

The speed to the third piece is 14 m/s at 45° east of north.

11.48. Model: The billiard balls will be modeled as particles. The two balls, m_1 (moving east) and m_2 (moving west), together are our system. This is an isolated system because any frictional force during the brief collision period is going to be insignificant. Within the impulse approximation, the momentum of our system will be conserved in the collision.

Visualize:



Note that $m_1 = m_2 = m$.

Solve: The equation $p_{\text{fx}} = p_{\text{ix}}$ yields:

$$m_1(v_{\text{fx}})_1 + m_2(v_{\text{fx}})_2 = m_1(v_{\text{ix}})_1 + m_2(v_{\text{ix}})_2 \quad \Rightarrow \quad m_1(v_\text{f})_1\cos\theta + 0 \text{ kg m/s} = m_1(v_{\text{ix}})_1 + m_2(v_{\text{ix}})_2$$

$$(v_\text{f})_1\cos\theta = (v_{\text{ix}})_1 + (v_{\text{ix}})_2 = 2.0 \text{ m/s} - 1.0 \text{ m/s} = 1.0 \text{ m/s}$$

The equation $p_{\text{fy}} = p_{\text{iy}}$ yields:

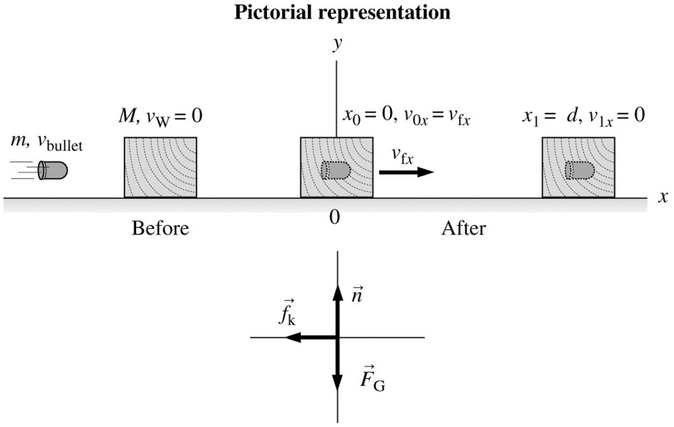
$$+m_1(v_{\text{fy}})_1\sin\theta + m_2(v_{\text{fy}})_2 = 0 \text{ kg m/s} \quad \Rightarrow \quad (v_\text{f})_1\sin\theta = -(v_{\text{fy}})_2 = -1.41 \text{ m/s}$$

$$(v_\text{f})_1 = \sqrt{(1.0 \text{ m/s})^2 + (-1.41 \text{ m/s})^2} = 1.7 \text{ m/s}, \; \theta = \tan^{-1}\left(\frac{1.41 \text{ m/s}}{1.0 \text{ m/s}}\right) = 55°$$

The angle is below +*x* axis, or south of east.

11.49. Model: Model the bullet and block as particles. This is an isolated system because any frictional force during the brief collision period is going to be insignificant. Within the impulse approximation, the momentum of our system will be conserved in the collision. After the collision, we will consider the frictional force and apply Newton's second law and kinematic equations to find the distance traveled by the block + bullet.

Visualize:



Solve: **(a)** Applying conservation of momentum to the collision gives

$$mv_{\text{bullet}} + Mv_W = (m+M)v_{fx} \quad\Rightarrow\quad v_{\text{bullet}} = \frac{m+M}{m}v_{fx}$$

The speed v_{ex} can be found from the kinematics equation

$$v_{1x}^2 = v_{0x}^2 + 2ad = v_{fx}^2 + 2ad \quad\Rightarrow\quad v_{fx} = \sqrt{-2ad}$$

The acceleration in the *x*-direction may be found using Newton's second law and the friction model. Because the block does not accelerate in the *y*-direction, the normal force must be the same magnitude as the force due to gravity (Newton's second law). Thus, the frictional force is $f_k = -\mu_k n = -\mu_k(m+M)g,$ where the negative sign indicates that the force acts in the negative *x*-direction. Newton's second law then gives the acceleration of the block as

$$a = F_{\text{net}}/(m+M) = -\mu_k(m+M)g/(m+M) = -\mu_k g$$

Inserting this into the expression for v_{ex} gives

$$v_{fx} = \sqrt{-2ad} = \sqrt{2\mu_k gd}$$

Finally, we insert this expression for v_{ex} into the expression for the bullet's velocity to find

$$v_{\text{bullet}} = \frac{m+M}{m}\sqrt{2\mu_k gd}$$

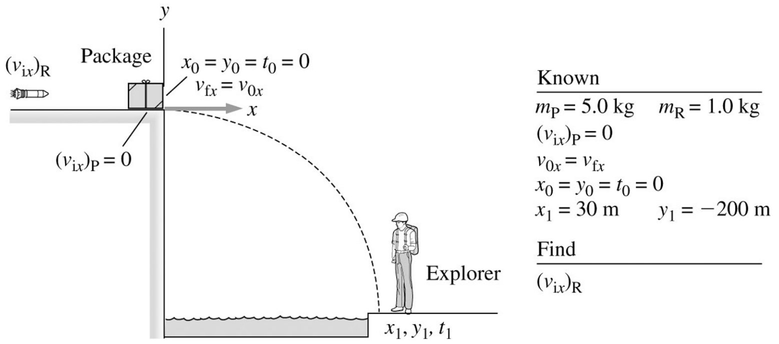
(b) Inserting the given quantities gives

$$v_{\text{bullet}} = \frac{0.010\text{ kg}+10\text{ kg}}{0.010\text{ kg}}\sqrt{2(0.20)(9.8\text{ m/s}^2)(0.050\text{ m})} = 4.4\times10^2\text{ m/s}$$

Assess: If we let the bullet's mass go to zero, we see that the bullet's speed goes to infinity, which is reasonable because a zero-mass bullet would need an infinite speed to make the block move. If the bullet's mass goes to infinity, the bullet's speed would go to $\sqrt{2\mu_k gd}$, which is just the result for the initial speed of an object that decelerates to a stop at a constant rate $(\mu_k g)$ over a distance *d*. In other words, the block becomes insignificant compared to the infinite-mass bullet.

11.50. **Model:** Model the package and the rocket as particles. This is a two-part problem. First we have an inelastic collision between the rocket (R) and the package (P). During the collision, momentum is conserved since no significant external force acts on the rocket and the package. However, as soon as the package + rocket system leaves the cliff they become a projectile motion problem.

Visualize:


Pictorial representation

Solve: The minimum velocity after collision that the package + rocket must have to reach the explorer is v_{0x}, which can be found as follows:

$$y_1 = y_0 + v_{0y}(t_1 - t_0) + \tfrac{1}{2}a_y(t_1 - t_0)^2 \quad \Rightarrow \quad -200\text{ m} = 0\text{ m} + 0\text{ m} + \tfrac{1}{2}(-9.8\text{ m/s}^2)t_1^2 \quad \Rightarrow \quad t_1 = 6.389\text{ s}$$

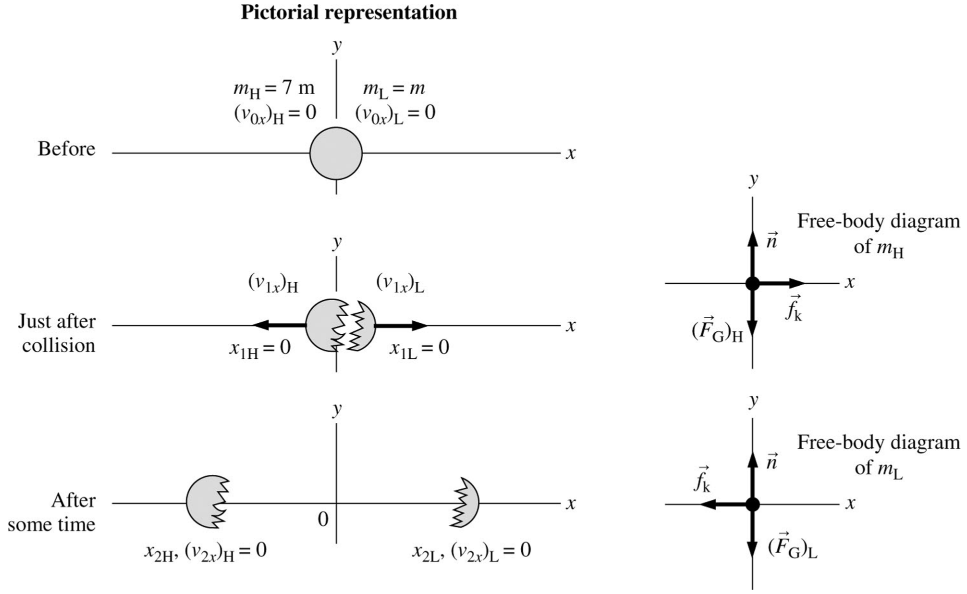
With this time, we can now find v_{0x} using $x_1 = x_0 + v_{0x}(t_1 - t_0) + \tfrac{1}{2}a_x(t_1 - t_0)^2$. We obtain

$$30\text{ m} = 0\text{ m} + v_{0x}(6.389\text{ s}) + 0\text{ m} \quad \Rightarrow \quad v_{0x} = 4.696\text{ m/s} = v_{fx}$$

We now use the momentum conservation equation $p_{fx} = p_{ix}$ which can be written

$$(m_R + m_P)v_{fx} = m_R(v_{ix})_R + m_P(v_{ix})_P$$
$$(1.0\text{ kg} + 5.0\text{ kg})(4.696\text{ m/s}) = (1.0\text{ kg})(v_{ix})_R + (5.0\text{ kg})(0\text{ m/s}) \quad \Rightarrow \quad (v_{ix})_R = 28\text{ m/s}$$

11.51. Model: This is a two-part problem. First, we have an explosion that creates two particles. The momentum of the system, comprising two fragments, is conserved in the explosion. Second, we will use kinematic equations and the model of kinetic friction to find the displacement of the lighter fragment.
Visualize:


Pictorial representation

Solve: The initial momentum is zero. Using momentum conservation $p_{\mathrm{f}x} = p_{\mathrm{i}x}$ during the explosion,

$$m_{\mathrm{H}}(v_{1x})_{\mathrm{H}} + m_{\mathrm{L}}(v_{1x})_{\mathrm{L}} = m_{\mathrm{H}}(v_{0x})_{\mathrm{H}} + m_{\mathrm{L}}(v_{0x})_{\mathrm{L}} \quad\Rightarrow\quad 7m(v_{1x})_{\mathrm{H}} + m(v_{1x})_{\mathrm{L}} = 0 \text{ kg m/s} \quad\Rightarrow\quad (v_{1x})_{\mathrm{H}} = -\left(\tfrac{1}{7}\right)(v_{1x})_{\mathrm{L}}$$

Because m_{H} slides to $x_{2\mathrm{H}} = -8.2$ m before stopping, we have

$$f_{\mathrm{k}} = \mu_{\mathrm{k}} n_{\mathrm{H}} = \mu_{\mathrm{k}} w_{\mathrm{H}} = \mu_{\mathrm{k}} m_{\mathrm{H}} g = m_{\mathrm{H}} a_{\mathrm{H}} \quad\Rightarrow\quad a_{\mathrm{H}} = \mu_{\mathrm{k}} g$$

Using kinematics,

$$(v_{2x})_{\mathrm{H}}^2 = (v_{1x})_{\mathrm{H}}^2 + 2a_{\mathrm{H}}(x_{2\mathrm{H}} - x_{1\mathrm{H}}) \quad\Rightarrow\quad 0\text{ m}^2/\text{s}^2 = \left(\tfrac{1}{7}\right)^2 (v_{1x})_{\mathrm{L}}^2 + 2\mu_{\mathrm{k}} g(-8.2\text{ m} - 0\text{ m})$$

$$(v_{1x})_{\mathrm{L}} = -88.74\sqrt{\mu_{\mathrm{k}}}\text{ m/s}$$

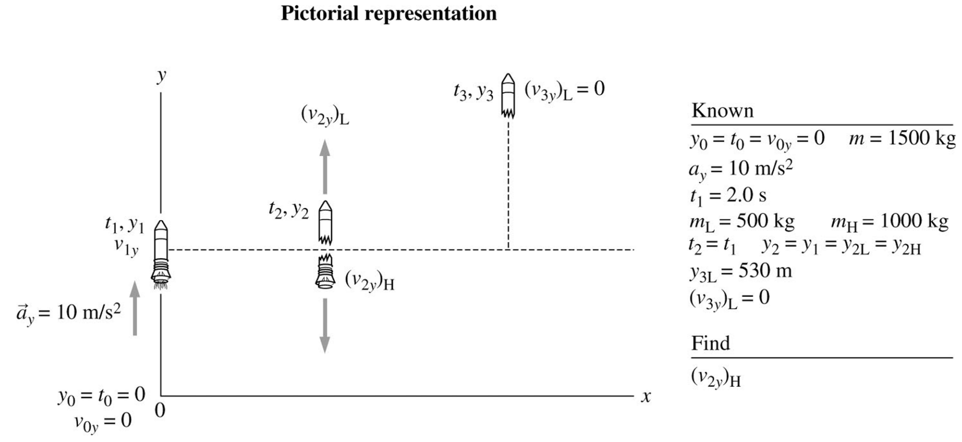
How far does m_{L} slide? Using the information obtained above in the following kinematic equation,

$$(v_{2x})_{\mathrm{L}}^2 = (v_{1x})_{\mathrm{L}}^2 + 2a_{\mathrm{L}}(x_{2\mathrm{L}} - x_{1\mathrm{L}}) \quad\Rightarrow\quad 0\text{ m}^2/\text{s}^2 = \mu_{\mathrm{k}}(88.74)^2 - 2\mu_{\mathrm{k}} g x_{2\mathrm{L}} \quad\Rightarrow\quad x_{2\mathrm{L}} = 4.0\times 10^2\text{ m}$$

Assess: Note that a_{H} is positive, but a_{L} is negative, and both are equal in magnitude to $\mu_{\mathrm{k}} g$. Also, $x_{2\mathrm{H}}$ is negative but $x_{2\mathrm{L}}$ is positive.

11.52. Model: We will model the two fragments of the rocket after the explosion as particles. We assume the explosion separates the two parts in a vertical manner. This is a three-part problem. In the first part, we will use kinematic equations to find the vertical position where the rocket breaks into two pieces. In the second part, we will apply conservation of momentum to the system (that is, the two fragments) in the explosion. In the third part, we will again use kinematic equations to find the velocity of the heavier fragment just after the explosion.

Visualize:


Pictorial representation
y
t_3, y_3
$(v_{3y})_{\mathrm{L}} = 0$
$(v_{2y})_{\mathrm{L}}$
t_2, y_2
t_1, y_1
v_{1y}
$(v_{2y})_{\mathrm{H}}$
$\vec{a}_y = 10\text{ m/s}^2$
$y_0 = t_0 = 0$
$v_{0y} = 0$
0
x
Known
$y_0 = t_0 = v_{0y} = 0 \quad m = 1500\text{ kg}$
$a_y = 10\text{ m/s}^2$
$t_1 = 2.0\text{ s}$
$m_{\mathrm{L}} = 500\text{ kg} \quad m_{\mathrm{H}} = 1000\text{ kg}$
$t_2 = t_1 \quad y_2 = y_1 = y_{2\mathrm{L}} = y_{2\mathrm{H}}$
$y_{3\mathrm{L}} = 530\text{ m}$
$(v_{3y})_{\mathrm{L}} = 0$
Find
$(v_{2y})_{\mathrm{H}}$

Solve: The rocket accelerates for 2.0 s from rest, so

$$v_{1y} = v_{0y} + a_y(t_1 - t_0) = 0\text{ m/s} + (10\text{ m/s}^2)(2.0\text{ s} - 0\text{ s}) = 20\text{ m/s}$$

$$y_1 = y_0 + v_{0y}(t_1 - t_0) + \tfrac{1}{2}a_y(t_1 - t_0)^2 = 0\text{ m} + 0\text{ m} + \tfrac{1}{2}(10\text{ m/s}^2)(2.0\text{ s})^2 = 20\text{ m}$$

At the explosion the equation $p_{\mathrm{f}y} = p_{\mathrm{i}y}$ is

$$m_{\mathrm{L}}(v_{2y})_{\mathrm{L}} + m_{\mathrm{H}}(v_{2y})_{\mathrm{H}} = (m_{\mathrm{L}} + m_{\mathrm{H}})v_{1y} \quad\Rightarrow\quad (500\text{ kg})(v_{2y})_{\mathrm{L}} + (1000\text{ kg})(v_{2y})_{\mathrm{H}} = (1500\text{ kg})(20\text{ m/s})$$

To find $(v_{2y})_{\mathrm{H}}$ we must first find $(v_{2y})_{\mathrm{L}}$, the velocity after the explosion of the upper section. Using kinematics,

$$(v_{3y})_{\mathrm{L}}^2 = (v_{2y})_{\mathrm{L}}^2 + 2(-9.8\text{ m/s}^2)(y_{3\mathrm{L}} - y_{2\mathrm{L}}) \quad\Rightarrow\quad (v_{2y})_{\mathrm{L}} = \sqrt{2(9.8\text{ m/s}^2)(530\text{ m} - 20\text{ m})} = 99.98\text{ m/s}$$

Now, going back to the momentum conservation equation we get

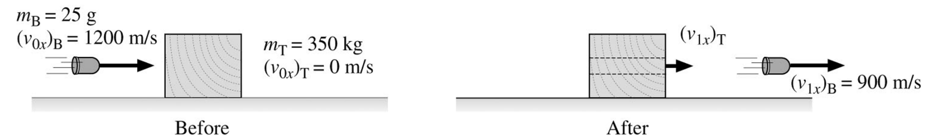
$$(500\text{ kg})(99.98\text{ m/s})+(1000\text{ kg})(v_{2y})_{\text{H}}=(1500\text{ kg})(20\text{ m/s})\quad\Rightarrow\quad(v_{2y})_{\text{H}}=-20\text{ m/s}$$

The negative sign indicates downward motion.

11.53. Model: Let the system be bullet + target. No external horizontal forces act on this system, so the horizontal momentum is conserved. Model the bullet and the target as particles. Since the target is much more massive than the bullet, it is reasonable to assume that the target undergoes no significant motion during the brief interval in which the bullet passes through it.

Visualize:

Pictorial representation



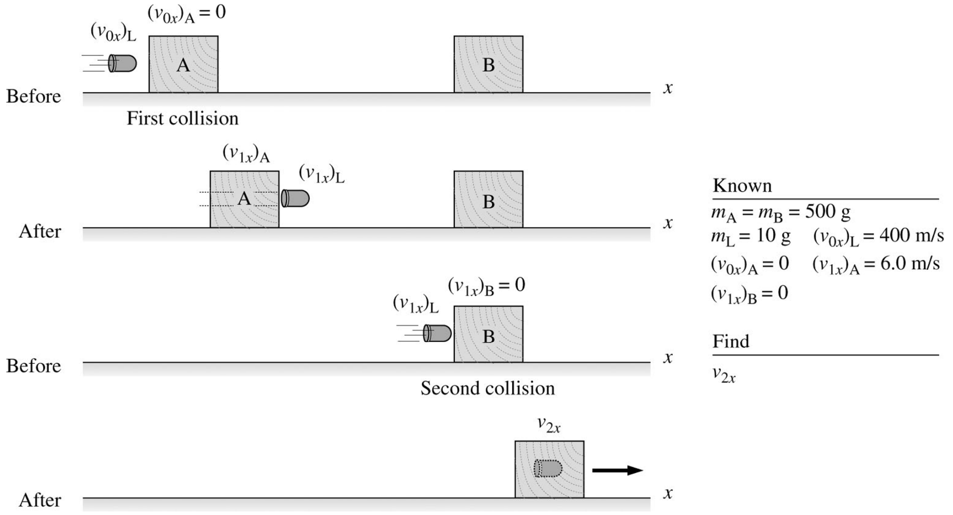
Solve: Use the conservation of momentum equation $p_{1x}=p_{0x}$ to find

$$m_{\text{T}}(v_{1x})_{\text{T}}+m_{\text{B}}(v_{1x})_{\text{B}}=m_{\text{T}}(v_{0x})_{\text{T}}+m_{\text{B}}(v_{0x})_{\text{B}}=0+m_{\text{B}}(v_{0x})_{\text{B}}$$

$$(v_{1x})_{\text{T}}=\frac{m_{\text{B}}(v_{0x})_{\text{B}}-m_{\text{B}}(v_{1x})_{\text{B}}}{m_{\text{T}}}=\frac{0.025\text{ kg}}{350\text{ kg}}[(1200\text{ m/s})-(900\text{ m/s})]=0.021\text{ m/s}$$

11.54. Model: Model the two blocks (A and B) and the bullet (L) as particles. This is a two-part problem. First, we have a collision between the bullet and the first block (A). Momentum is conserved since no external force acts on the system (bullet + block A). The second part of the problem involves a perfectly inelastic collision between the bullet and block B. Momentum is again conserved for this system (bullet + block B).

Visualize:



Solve: For the first collision the equation $p_{fx} = p_{ix}$ is

$$m_L(v_{1x})_L + m_A(v_{1x})_A = m_L(v_{0x})_L + m_A(v_{0x})_A$$

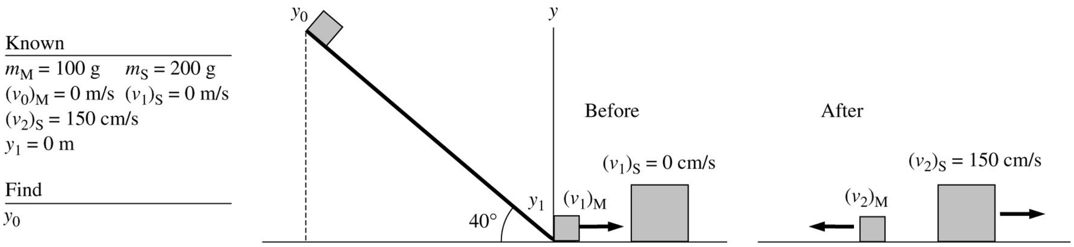
$$(0.010\text{ kg})(v_{1x})_L + (0.500\text{ kg})(6.0\text{ m/s}) = (0.010\text{ kg})(400\text{ m/s}) + 0\text{ kg m/s} \quad\Rightarrow\quad (v_{1x})_L = 100\text{ m/s}$$

The bullet emerges from the first block at 100 m/s. For the second collision the equation $p_{fx} = p_{ix}$ is

$$(m_L + m_B)v_{2x} = m_L(v_{1x})_L \quad\Rightarrow\quad (0.010\text{ kg} + 0.500\text{ kg})v_{2x} = (0.010\text{ kg})(100\text{ m/s}) \quad\Rightarrow\quad v_{2x} = 2.0\text{ m/s}$$

11.55. Model: Model the marble and the steel ball as particles. We will assume an elastic collision between the marble and the ball, and apply the conservation of momentum and the conservation of energy equations. We will also assume zero rolling friction between the marble and the incline.

Visualize:



This is a two-part problem. In the first part, we will apply the conservation of energy equation to find the marble's speed as it exits onto a horizontal surface. We have put the origin of our coordinate system on the horizontal surface just where the marble exits the incline. In the second part, we will consider the elastic collision between the marble and the steel ball.

Solve: The conservation of energy equation $K_1 + U_{g1} = K_0 + U_{g0}$ gives us:

$$\frac{1}{2}m_M(v_1)_M^2 + m_M g y_1 = \frac{1}{2}m_M(v_0)_M^2 + m_M g y_0$$

Using $(v_0)_M = 0$ m/s and $y_1 = 0$ m, we get $\frac{1}{2}(v_1)_M^2 = gy_0 \Rightarrow (v_1)_M = \sqrt{2gy_0}$. When the marble collides with the steel ball, the elastic collision gives the ball velocity

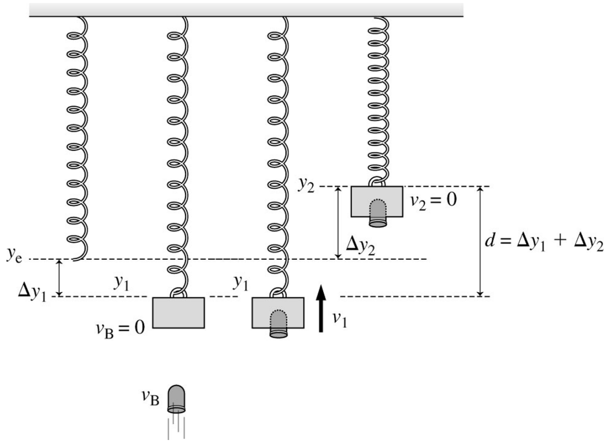
$$(v_2)_S = \frac{2m_M}{m_M + m_S}(v_1)_M = \frac{2m_M}{m_M + m_S}\sqrt{2gy_0}$$

Solving for y_0 gives

$$y_0 = \frac{1}{2g}\left[\frac{m_M + m_S}{2m_M}(v_2)_S\right]^2 = 0.258\text{ m} = 25.8\text{ cm}$$

11.56. Model: Assume an ideal spring that obeys Hooke's law. There is no friction, hence the mechanical energy $K + U_g + U_s$ is conserved.

Visualize:



We have chosen to place the origin of the coordinate system on the free end of the spring that is neither stretched nor compressed, that is, at the equilibrium position of the end of the unstretched spring. The bullet's mass is m and the block's mass is M.

Solve: (a) The energy conservation equation $K_2 + U_{s2} + U_{g2} = K_1 + U_{s1} + U_{g1}$ becomes

$$\frac{1}{2}(m+M)v_2^2 + \frac{1}{2}k(y_2 - y_e)^2 + (m+M)g(y_2 - y_e) = \frac{1}{2}(m+M)v_1^2 + \frac{1}{2}k(y_1 - y_e)^2 - (m+M)g(y_1 - y_e)$$

Noting $v_2 = 0$ m/s, we can rewrite the above equation as

$$k(\Delta y_2)^2 + 2(m+M)g(\Delta y_2 + \Delta y_1) = (m+M)v_1^2 + k(\Delta y_1)^2$$

Let us express v_1 in terms of the bullet's initial speed v_B by using the momentum conservation equation $p_f = p_i$ which is $(m+M)v_1 = mv_B + Mv_{block}$. Since $v_{block} = 0$ m/s, we have

$$v_1 = \left(\frac{m}{m+M}\right)v_B$$

We can also find the magnitude of y_1 from the equilibrium condition $k(y_1 - y_e) = Mg.$

$$\Delta y_1 = \frac{Mg}{k}$$

With these substitutions for v_1 and Δy_1, the energy conservation equation simplifies to

$$k(\Delta y_2)^2 + 2(m+M)g(\Delta y_1 + \Delta y_2) = \frac{m^2 v_B^2}{(m+M)} + k\left(\frac{Mg}{k}\right)^2$$

$$\Rightarrow v_B^2 = 2\left(\frac{m+M}{m}\right)^2 g(\Delta y_1 + \Delta y_2) - \frac{(m+M)}{m^2}\frac{M^2 g^2}{k} + k\left(\frac{m+M}{m^2}\right)(\Delta y_2)^2$$

We still need to include the spring's maximum compression (d) into this equation. We assume that $d = \Delta y_1 + \Delta y_2$, that is, maximum compression is measured from the initial position (y_1) of the block. Thus, using $\Delta y_2 = d - \Delta y_1 = (d - Mg/k)$, we have

$$v_B = \left[2\left(\frac{m+M}{m}\right)^2 gd - \left(\frac{m+M}{m^2}\right)\frac{M^2 g^2}{k} + k\left(\frac{m+M}{m^2}\right)(d - Mg/k)^2\right]^{1/2}$$

(b) Using $m = 0.010$ kg, $M = 2.0$ kg, $k = 50$ N/m, and $d = 0.45$ m,

$$v_\text{B}^2 = 2\left(\frac{2.010\text{ kg}}{0.010\text{ kg}}\right)^2 (9.8\text{ m/s}^2)(0.45\text{ m}) - (2.010\text{ kg})\left(\frac{2.0\text{ kg}}{0.010\text{ kg}}\right)^2 (9.8\text{ m/s}^2)^2/(50\text{ N/m})$$

$$+(50\text{ N/m})(2.010\text{ kg})\frac{1}{(0.010\text{ kg})^2}[0.45\text{ m} - (2.0\text{ kg})\times(9.8\text{ m/s}^2)/50\text{ N/m}]^2$$

$$\Rightarrow v_\text{B} = 453\text{ m/s}$$

The bullet has a speed of 4.5×10^2 m/s.

Assess: This is a reasonable speed for the bullet.

11.57. Model: The track is frictionless.

Visualize: $v_\text{c} = \sqrt{Rg}$

Solve: (a) First use conservation of momentum during the collision, then conservation of energy as the combined block goes to the top of the loop.

$$\Sigma\vec{p}_\text{i} = \Sigma\vec{p}_\text{f}$$

$$mv_m + 0 = (m+M)v_\text{tot}$$

$$v_m = \frac{m+M}{m}v_\text{tot}$$

Now use the conservation of energy. v_tot is the speed of the combined block just after the collision.

$$U_\text{i} + K_\text{i} = U_\text{f} + K_\text{f}$$

$$0 + \frac{1}{2}(m+M)v_\text{tot}^2 = (m+M)g(2R) + \frac{1}{2}(m+M)(v_\text{c})^2$$

Cancel $(m+M)$ and replace v_c with $\sqrt{Rg}$.

$$v_\text{tot}^2 = 4Rg + Rg = 5Rg \Rightarrow v_\text{tot} = \sqrt{5Rg}$$

$$v_m = \frac{m+M}{m}v_\text{tot} = \frac{m+M}{m}\sqrt{5Rg}$$

(b) First use conservation of momentum and kinetic energy during the collision, then conservation of mechanical energy as the big block goes to the top of the loop. Call the speed of M just after the elastic collision V and the speed of m just after the collision v_m'.

$$\Sigma\vec{p}_\text{i} = \Sigma\vec{p}_\text{f}$$

We drop the vectors because this is one-dimensional motion, but v' may be negative.

$$mv_m + 0 = mv' + MV$$

$$v_m = v' + \frac{M}{m}V$$

Now use the conservation of energy as the block goes to the top of the loop.

$$U_\text{i} + K_\text{i} = U_\text{f} + K_\text{f}$$

$$0 + \frac{1}{2}mv_m'^2 + \frac{1}{2}MV^2 = Mg(2R) + \frac{1}{2}mv_m'^2 + \frac{1}{2}M\left(\sqrt{Rg}\right)^2$$

Subtract $\frac{1}{2}mv_m'^2$ from both sides, cancel M, and solve for V.

$$V = \sqrt{4Rg + Rg} = \sqrt{5Rg}$$

Now go back to the conservation of kinetic energy in the elastic collision.

$$\sum K_\text{i} = \sum K_\text{f}$$

$$\frac{1}{2}mv_m^2 = \frac{1}{2}mv_m'^2 + \frac{1}{2}MV^2$$

Cancel $\frac{1}{2}$ and divide by m.

$$v_m^2 = v_m'^2 + \frac{M}{m}5Rg$$

Find $v_m'^2$ from the momentum equation.

$$v_m^2 = \left(v_m - \frac{M}{m}V\right)^2 + \frac{M}{m}5Rg$$

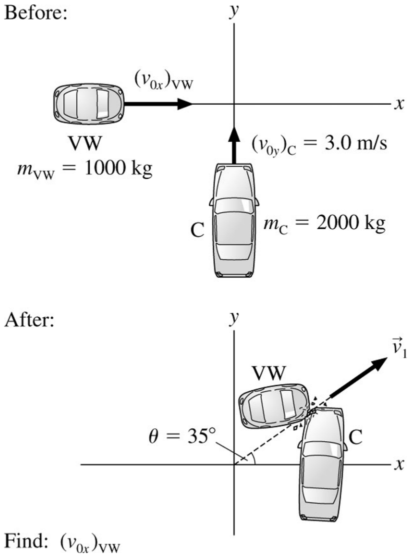
$$v_m^2 = v_m^2 - 2\frac{M}{m}v_mV + \left(\frac{M}{m}V\right)^2 + \frac{M}{m}5Rg$$

Subtract v_m^2 from both sides, cancel $\frac{M}{m}$, and solve for v_m.

$$v_m = \frac{1}{2}\frac{M}{m}V + \frac{1}{2}\frac{5Rg}{\sqrt{5Rg}} = \frac{1}{2}\frac{M}{m}\sqrt{5Rg} + \frac{1}{2}\sqrt{5Rg} = \frac{1}{2}\frac{m+M}{m}\sqrt{5Rg}$$

Assess: We expected the initial speed needed to be greater for the inelastic case because kinetic energy isn't conserved in the collision.

11.58. Model: This is an inelastic collision. The total momentum of the Volkswagen + Cadillac system is conserved.
Visualize:



Solve: Apply conservation of momentum in the x- and y-directions.

$$(m_\text{C} + m_\text{VW})v_{1x} = (m_\text{C} + m_\text{VW})v_1\cos\theta = m_\text{VW}(v_{0x})_\text{VW}$$

$$(m_\text{C} + m_\text{VW})v_{1y} = (m_\text{C} + m_\text{VW})v_1\sin\theta = m_\text{C}(v_{0y})_\text{C}$$

From the y-equation, we find

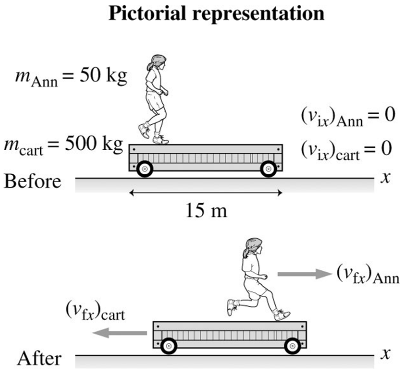
$$v_1 = \frac{m_C (v_{0y})_C}{(m_C + m_{VW})\sin\theta} = \frac{(2000\text{ kg})(3.0\text{ m/s})}{(3000\text{ kg})\sin 35°} = 3.49\text{ m/s}$$

Inserting this value into the x-equation gives

$$(v_{0x})_{VW} = \frac{(m_C + m_{VW}) v_1 \cos\theta}{m_{VW}} = \frac{(3000\text{ kg})(3.49\text{ m/s})\cos 35°}{1000\text{ kg}} = 8.6\text{ m/s}$$

11.59. Model: Model Ann and cart as particles. The initial momentum is $p_i = 0$ kg m/s in a coordinate system attached to the ground. As Ann begins running to the right, the cart will have to recoil to the left to conserve momentum.

Visualize:



Solve: The difficulty with this problem is that we are given Ann's velocity of 5.0 m/s relative to the cart. If the cart is also moving with velocity v_{cart} then Ann's velocity relative to the ground is not 5.0 m/s. Using the Galilean transformation equation for velocity, Ann's velocity relative to the ground is

$$(v_{fx})_{Ann} = (v_{fx})_{cart} + 5.0\text{ m/s}$$

Now, the momentum conservation equation $p_{ix} = p_{fx}$ is

$$0\text{ kg m/s} = m_{Ann}(v_{fx})_{Ann} + m_{cart}(v_{fx})_{cart} \quad \Rightarrow \quad 0\text{ kg m/s} = (50\text{ kg})[(v_{fx})_{cart} + 5.0\text{ m/s}] + (500\text{ kg})(v_{fx})_{cart}$$

$$(v_{fx})_{cart} = -0.45\text{ m/s}$$

Using the recoil velocity $(v_{fx})_{cart}$ relative to the ground, we find Ann's velocity relative to the ground to be

$$(v_{fx})_{Ann} = 5.00\text{ m/s} - 0.45\text{ m/s} = 4.55\text{ m/s}$$

The distance Ann runs relative to the ground is $\Delta x = (v_{fx})_{Ann}\Delta t$, where Δt is the time it takes to reach the end of the cart. Relative to the cart, which is 15 m long, Ann's velocity is 5 m/s. Thus, $\Delta t = (15\text{ m})/(5.0\text{ m/s}) = 3.0\text{ s}$. Her distance over the ground during this interval is

$$\Delta x = (v_{fx})_{Ann}\Delta t = (4.55\text{ m/s})(3.0\text{ s}) = 14\text{ m}$$

11.60. Model: Change in momentum is given by the impulse-momentum theorem (Equation 11.8).

Solve: Using $\Delta p = mv_{\text{f}x} - mv_{\text{i}x} = J_x = \int_{t_\text{i}}^{t_\text{f}} F_x(t)dx$ with $v_{\text{i}x} = 0,$ the velocity after the force has been applied is

$$\begin{aligned} v_{\text{f}x} &= \frac{1}{m}\int_{t_\text{i}}^{t_\text{f}} F_x(t)dx = \frac{10\text{ N}}{0.25\text{ kg}}\int_{0.0\text{ s}}^{2.0\text{ s}} \sin(2\pi t/4.0\text{ s})dt \\ &= (40\text{ N/kg})\left[-\frac{4.0\text{ s}}{2\pi}\cos\left(\frac{2\pi t}{4.0\text{ s}}\right)\Bigg|_{0.0\text{ s}}^{2.0\text{ s}}\right] \\ &= -(25.5\text{ m/s})[\cos(\pi) - \cos(0)] \\ &= -(25.5\text{ m/s})(-1-1) \\ &= 51\text{ m/s} \end{aligned}$$

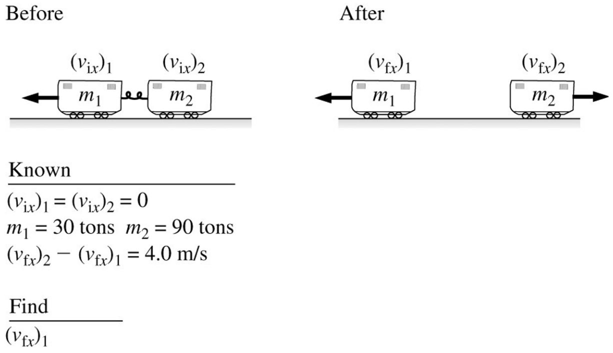
Assess: The force is applied for half the period of 4.0 s. During that time, $\sin\left(\frac{2\pi t}{4.0\text{ s}}\right)$ is positive, so an object initially at rest acquires a positive velocity.

11.61. Solve: Apply the impulse-momentum theorem (Equation 11.8) to find the initial velocity:

$$\begin{aligned} \Delta p &= \int F dt \\ v_{\text{f}x} &= v_{\text{i}x} + \frac{1}{m}\int F dt \\ &= -5.0\text{ m/s} + \frac{1}{0.500\text{ kg}}\int_{-2}^{2}(4 - t^2)dt \\ &= -5.0\text{ m/s} + \frac{1}{0.500\text{ kg}}\left(4t - \frac{1}{3}t^3\right)_{-2}^{2} \\ &= -5.0\text{ m/s} + \frac{1}{0.500\text{ kg}}\left[8 - \frac{8}{3} - \left(-8 + \frac{8}{3}\right)\right] \\ &= 16\text{ m/s} \end{aligned}$$

11.62. Model: The two railcars make up a system. The impulse approximation is used while the spring is expanding, so friction can be ignored.
Visualize:

Pictorial representation



Solve: Since the cars are at rest initially, the total momentum of the system is zero. Conservation of momentum gives

$$0 = m_1(v_{fx})_1 + m_2(v_{fx})_2$$

We are only told that the relative velocity of the two cars after the spring expands is 4.0 m/s, so

$$(v_{fx})_2 - (v_{fx})_1 = 4.0 \text{ m/s}$$

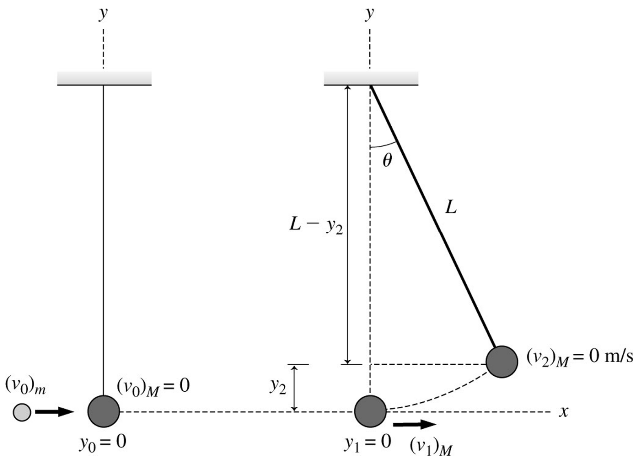
Substitute $(v_{fx})_2 = (v_{fx})_1 + 4.0$ m/s into the conservation of momentum equation, then solve for $(v_{fx})_1$:

$$0 = m_1(v_{fx})_1 + m_2[(v_{fx})_1 + 4.0 \text{ m/s}]$$

$$(v_{fx})_1 = -\frac{m_2(4.0 \text{ m/s})}{(m_1 + m_2)} = -\frac{(90 \text{ tons})(4.0 \text{ m/s})}{(30 \text{ tons} + 90 \text{ tons})} = -3.0 \text{ m/s}$$

so the speed of the 30 ton car relative to the ground is 3.0 m/s.
Assess: The other more massive railcar has a velocity $(v_{fx})_2 = (v_{fx})_1 + 4.0 \text{ m/s} = 1.0 \text{ m/s}$. A slower speed for the more massive car makes sense.

11.63. Model: We can divide this problem into two parts. First, we have an elastic collision between the 20 g ball (m) and the 100 g ball (M). Second, the 100 g ball swings up as a pendulum.
Visualize:



The figure shows three distinct moments of time: the time before the collision, the time after the collision but before the two balls move, and the time the 100 g ball reaches its highest point. We place the origin of our coordinate system on the 100 g ball when it is hanging motionless.
Solve: For a perfectly elastic collision, the ball moves forward with speed

$$(v_1)_M = \frac{2m_m}{m_m + m_M}(v_0)_m = \frac{1}{3}(v_0)_m$$

In the second part, the sum of the kinetic and gravitational potential energy is conserved as the 100 g ball swings up after the collision. That is, $K_2 + U_{g2} = K_1 + U_{g1}$. We have

$$\frac{1}{2}M(v_2)_M^2 + Mgy_2 = \frac{1}{2}M(v_1)_M^2 + Mgy_1$$

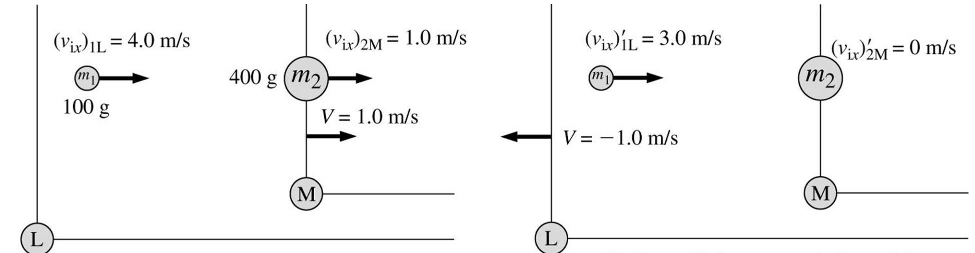
Using $(v_2)_M = 0 \text{ J}, (v_1)_M = \frac{(v_0)_m}{3}, y_1 = 0 \text{ m}$, and $y_2 = L - L\cos\theta$, the energy equation simplifies to

$$g(L - L\cos\theta) = \frac{1}{2}\frac{(v_0)_m^2}{9}$$

$$\Rightarrow (v_0)_m = \sqrt{18 \text{ g } L(1-\cos\theta)} = \sqrt{18(9.8 \text{ m/s}^2)(1.0 \text{ m})(1-\cos 50°)} = 7.9 \text{ m/s}$$

11.64. Model: Model the balls as particles. We will use the Galilean transformation of velocities (Equation 11.30) to analyze the problem of elastic collisions. We will transform velocities from the lab frame L to a frame M in which one ball is at rest. This allows us to apply Equation 11.30 to a perfectly elastic collision in M. After finding the final velocities of the balls in M, we can then transform these velocities back to the lab frame L.

Visualize: Let M be the frame of the 400 g ball. Denoting masses as $m_1 = 100$ g and $m_2 = 400$ g, the initial velocities in the S frame are $(v_{ix})_{1L} = +4.0$ m/s and $(v_{ix})_{2L} = +1.0$ m/s.

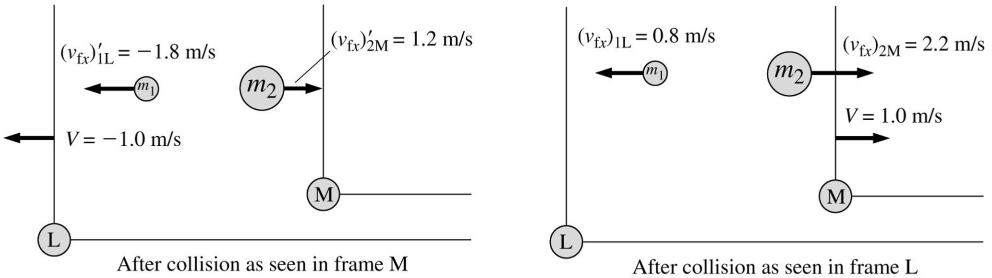


Before collision as seen in frame L

(a)

Before collision as seen in frame M

(b)


$(v_{fx})'_{1L} = -1.8$ m/s
m_1
$V = -1.0$ m/s
$(v_{fx})'_{2M} = 1.2$ m/s
m_2
M
L
$(v_{fx})_{1L} = 0.8$ m/s
m_1
$(v_{fx})_{2M} = 2.2$ m/s
m_2
$V = 1.0$ m/s
M
L

After collision as seen in frame M

(c)

After collision as seen in frame L

(d)

Figures **(a)** and **(b)** show the before-collision situations in frames L and M, respectively. The after-collision velocities in M are shown in figure **(c)**. Figure **(d)** indicates velocities in L after they have been transformed to L from M.

Solve: In frame L, $(v_{ix})_{1L} = 4.0$ m/s and $(v_{ix})_{2L} = 1.0$ m/s. Because M is the reference frame of the 400 g ball, $(v_x)_{ML} = 1.0$ m/s. The velocities of the two balls in this frame can be obtained using the Galilean transformation of velocities $(v_x)_{OM} = (v_x)_{OL} - (v_x)_{ML}$. So,

$$(v_{ix})_{1M} = (v_{ix})_{1L} - (v_x)_{ML} = 4.0\text{ m/s} - 1.0\text{ m/s} = 3.0\text{ m/s} \qquad (v_{ix})_{2M} = (v_{ix})_{2L} - (v_x)_{ML} = 1.0\text{ m/s} - 1.0\text{ m/s} = 0\text{ m/s}$$

Figure **(b)** shows the "before" situation in frame M where the ball 2 is at rest.

Now we can use Equation 11.30 to find the after-collision velocities in frame M.

$$(v_{fx})_{1M} = \frac{m_1 - m_2}{m_1 + m_2}(v_{ix})_{1M} = \frac{100\text{ g} - 400\text{ g}}{100\text{ g} + 400\text{ g}}(3.0\text{ m/s}) = -1.80\text{ m/s}$$

$$(v_{fx})_{2M} = \frac{2m_1}{m_1 + m_2}(v_{ix})_{1M} = \frac{2(100\text{ g})}{100\text{ g} + 400\text{ g}}(3.0\text{ m/s}) = 1.20\text{ m/s}$$

Finally, we need to apply the reverse Galilean transformation $(v_x)_{OM} = (v_x)_{OL} + (v_x)_{LM}$ with the same $(v_x)_{LM}$, to transform the after-collision velocities back to the lab frame L.

$$(v_{fx})_{1L} = (v_{fx})_{1M} + (v_x)_{ML} = -1.80\text{ m/s} + 1.0\text{ m/s} = -0.80\text{ m/s}$$

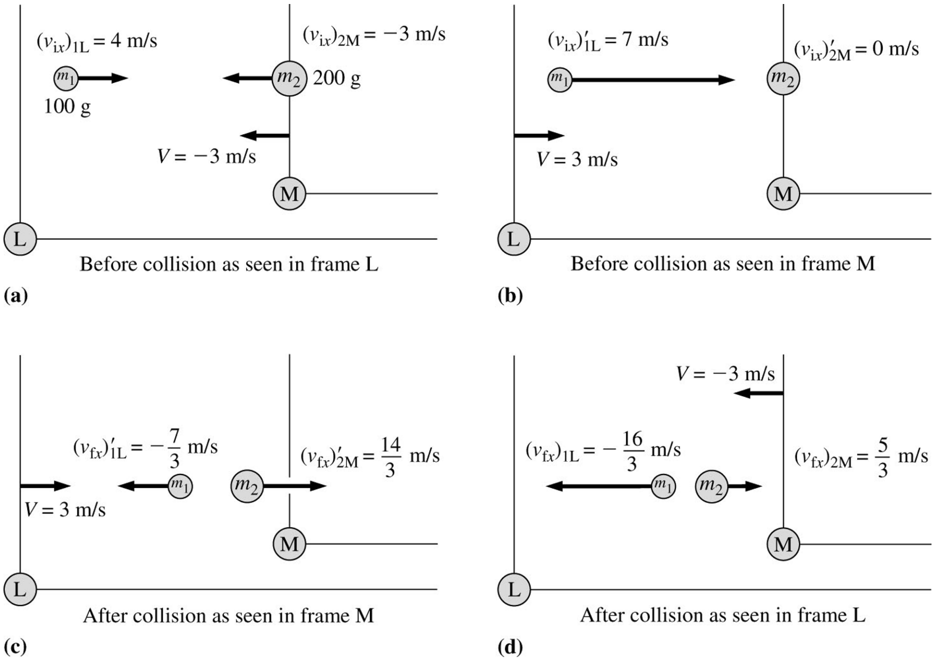
$$(v_{fx})_{2L} = (v_{fx})_{2M} + (v_x)_{ML} = 1.20\text{ m/s} + 1.0\text{ m/s} = 2.20\text{ m/s}$$

Figure **(d)** shows the "after" situation in frame L. The 100 g ball moves left at 0.80 m/s, the 400 g ball right at 2.2 m/s.

Assess: The magnitudes of the after-collision velocities are similar to the magnitudes of the before-collision velocities.

11.65. Model: Model the balls as particles. We will use the Galilean transformation of velocities to analyze the problem of elastic collisions. We will transform velocities from the lab frame L to a frame M in which one ball is at rest. This allows us to apply equations to the case of a perfectly elastic collision in M, find the final velocities of the balls in M, and then transform these velocities back to the lab frame L.

Visualize: Let M be the frame of the 200 g ball. Denoting masses as $m_1 = 100$ g and $m_2 = 200$ g, the initial velocities in the L frame are $(v_{ix})_{1L} = 4$ m/s and $(v_{ix})_{2L} = -3$ m/s.



$(v_{ix})_{1L} = 4$ m/s
$(v_{ix})_{2M} = -3$ m/s
m_1
100 g
m_2
200 g
$V = -3$ m/s
M
L
Before collision as seen in frame L
(a)
$(v_{ix})'_{1L} = 7$ m/s
$(v_{ix})'_{2M} = 0$ m/s
$V = 3$ m/s
Before collision as seen in frame M
(b)
$(v_{fx})'_{1L} = -\frac{7}{3}$ m/s
$(v_{fx})'_{2M} = \frac{14}{3}$ m/s
$V = 3$ m/s
After collision as seen in frame M
(c)
$V = -3$ m/s
$(v_{fx})_{1L} = -\frac{16}{3}$ m/s
$(v_{fx})_{2M} = \frac{5}{3}$ m/s
After collision as seen in frame L
(d)

Figure **(a)** shows the before-collision situation as seen in frame L, and figure **(b)** shows the before-collision situation as seen in frame M. The after-collision velocities in M are shown in figure **(c)**, and figure **(d)** indicates velocities in L after they have been transformed to frame L from M.

Solve: (a) In the L frame, $(v_{ix})_{1L} = 4$ m/s and $(v_{ix})_{2L} = -3$ m/s. M is the reference frame of the 200 g ball, so $(v_x)_{ML} = -3$ m/s. The velocities of the two balls in this frame can be obtained using the Galilean transformation of velocities $(v_x)_{OM} = (v_x)_{OL} + (v_x)_{LM}$. So,

$$(v_{ix})_{1M} = (v_{ix})_{1L} - (v_{ix})_{ML} = 4\text{ m/s} - (-3\text{ m/s}) = 7\text{ m/s} \qquad (v_{ix})_{2M} = (v_{ix})_{2L} - (v_{ix})_{ML} = -3\text{ m/s} - (-3\text{ m/s}) = 0\text{ m/s}$$

Figure **(b)** shows the "before" situation, where ball 2 is at rest.

Now we can use Equation 11.29 to find the after-collision velocities in frame M:

$$(v_{fx})_{1M} = \frac{m_1 - m_2}{m_1 + m_2}(v_{ix})_{1M} = \frac{100\text{ g} - 200\text{ g}}{100\text{ g} + 200\text{ g}}(7\text{ m/s}) = -\frac{7}{3}\text{ m/s}$$

$$(v_{fx})_{2M} = \frac{2m_1}{m_1 + m_2}(v_{ix})_{1M} = \frac{2(100)\text{ g}}{100\text{ g} + 200\text{ g}}(7\text{ m/s}) = \frac{14}{3}\text{ m/s}$$

Finally, we need to apply the reverse Galilean transformation $(v_x)_{OM} = (v_x)_{OL} + (v_x)_{LM}$ with the same $(v_x)_{LM}$, to transform the after-collision velocities back to the lab frame S:

$$(v_{fx})_{1L} = (v_{fx})_{1M} + (v_x)_{ML} = -\frac{7}{3}\text{m/s} - 3\text{ m/s} = -5.33\text{ m/s}$$

$$(v_{fx})_{2L} = (v_{fx})_{2M} + (v_x)_{ML} = \frac{14}{3}\text{m/s} - 3\text{ m/s} = 1.67\text{ m/s}$$

Figure **(d)** shows the "after" situation in the lab frame. The 100 g ball is moving left at 5.3 m/s; the 200 g ball is moving right at 1.7 m/s.

(b) The momentum conservation equation $p_{\text{f}x} = p_{\text{i}x}$ for a perfectly inelastic collision is

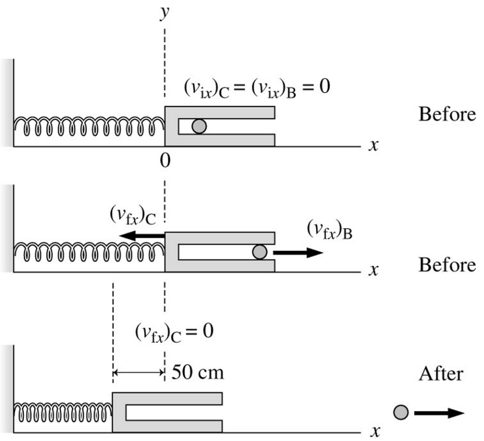
$$(m_1 + m_2)v_{\text{f}x} = m_1(v_{\text{i}x})_1 + m_2(v_{\text{i}x})_2$$

$$(0.100\text{ kg} + 0.200\text{ kg})v_{\text{f}x} = (0.100\text{ kg})(4.0\text{ m/s}) + (0.200\text{ kg})(-3.0\text{ m/s}) \Rightarrow v_{\text{f}x} = -0.667\text{ m/s}$$

Both balls are moving left at 0.67 m/s.

11.66. Model: Assume an ideal, massless spring that obeys Hooke's law. Let us also assume that the cannon (C) fires balls (B) horizontally and that the spring is directly behind the cannon to absorb all motion.

Visualize:


y
$(v_{\text{i}x})_{\text{C}} = (v_{\text{i}x})_{\text{B}} = 0$
Before
0
x
$(v_{\text{f}x})_{\text{C}}$
$(v_{\text{f}x})_{\text{B}}$
x
Before
$(v_{\text{f}x})_{\text{C}} = 0$
50 cm
After
x

The before-and-after pictorial representation is shown, with the origin of the coordinate system located at the spring's free end when the spring is neither compressed nor stretched. This free end of the spring is just behind the cannon.

Solve: The momentum conservation equation $p_{\text{f}x} = p_{\text{i}x}$ is

$$m_{\text{B}}(v_{\text{f}x})_{\text{B}} + m_{\text{C}}(v_{\text{f}x})_{\text{C}} = m_{\text{B}}(v_{\text{i}x})_{\text{B}} + m_{\text{C}}(v_{\text{i}x})_{\text{C}}$$

Since the initial momentum is zero,

$$(v_{\text{f}x})_{\text{B}} = -\frac{m_{\text{C}}}{m_{\text{B}}}(v_{\text{f}x})_{\text{C}} = -\left(\frac{200\text{ kg}}{10\text{ kg}}\right)(v_{\text{f}x})_{\text{C}} = -20(v_{\text{f}x})_{\text{C}}$$

The mechanical energy conservation equation for the cannon + spring $K_{\text{f}} + U_{\text{sf}} = K_{\text{i}} + U_{\text{si}}$ is

$$\frac{1}{2}m(v_{\text{f}})_{\text{C}}^2 + \frac{1}{2}k(\Delta x)^2 = \frac{1}{2}m(v_{\text{i}})_{\text{C}}^2 + 0\text{ J} \Rightarrow 0\text{ J} + \frac{1}{2}k(\Delta x)^2 = \frac{1}{2}m(v_{\text{f}x})_{\text{C}}^2$$

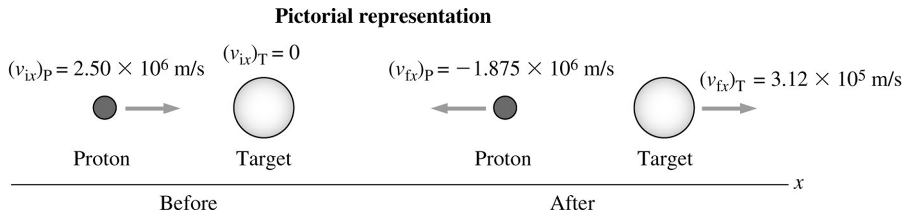
$$\Rightarrow (v_{\text{f}x})_{\text{C}} = \pm\sqrt{\frac{k}{m}}\Delta x = \pm\sqrt{\frac{(20{,}000\text{ N/m})}{200\text{ kg}}}(0.50\text{ m}) = \pm 5.0\text{ m/s}$$

To make this velocity physically correct, we retain the minus sign with $(v_{\text{f}x})_{\text{C}}$. Substituting into the momentum conservation equation yields:

$$(v_{\text{f}x})_{\text{B}} = -20(-5.0\text{ m/s}) = 100\text{ m/s}$$

11.67. Model: Model the proton (P) and the target nucleus (T) as particles. The proton and the target nucleus make our system and in the collision between them momentum is conserved. This is due to the impulse approximation because the collision lasts a very short time and the external forces acting on the system during this time are not significant.

Visualize:


Pictorial representation
$(v_{ix})_P = 2.50 \times 10^6$ m/s
$(v_{ix})_T = 0$
$(v_{fx})_P = -1.875 \times 10^6$ m/s
$(v_{fx})_T = 3.12 \times 10^5$ m/s
Proton
Target
Proton
Target
x
Before
After

Solve: The conservation of momentum equation $p_{fx} = p_{ix}$ gives

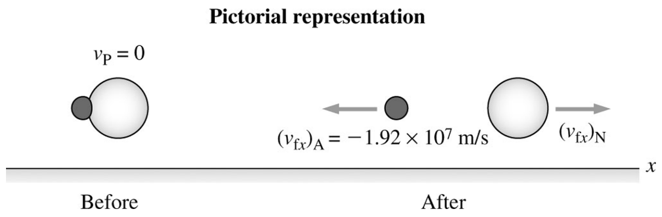
$$m_T(v_{fx})_T + m_P(v_{fx})_P = m_T(v_{ix})_T + m_P(v_{ix})_P$$

$$m_T(3.12\times10^5 \text{ m/s}) + (1 \text{ u})(-0.750\times2.50\times10^6 \text{ m/s}) = 0 \text{ u m/s} + (1 \text{ u})(2.50\times10^6 \text{ m/s})$$

$$m_T = 14.0 \text{ u}$$

Assess: This is the mass of the nucleus of a nitrogen atom.

11.68. Model: This problem deals with an "explosion" in which a ^{214}Po nucleus (P) decays into an alpha-particle (A) and a daughter nucleus (N). During the "explosion" or decay, the total momentum of the system is conserved.
Visualize:

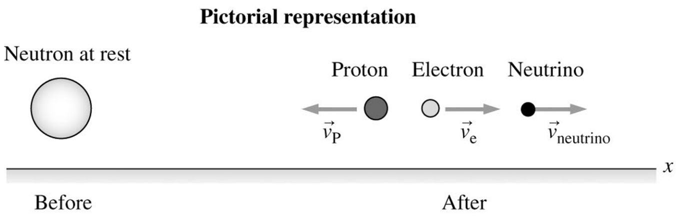


Solve: Conservation of mass requires the daughter nucleus to have mass $m_N = 214 \text{ u} - 4 \text{ u} = 210$ u. The conservation of momentum equation $p_{fx} = p_{ix}$ gives

$$m_N(v_{fx})_N + m_A(v_{fx})_A = (m_N + m_A)v_P \quad \Rightarrow \quad (210 \text{ u})(v_{fx})_N + (4 \text{ u})(-1.92\times10^7 \text{m/s}) = 0 \text{ u m/s}$$

$$(v_{fx})_N = 3.66\times10^5 \text{ m/s}$$

11.69. Model: The neutron's decay is an "explosion" of the neutron into several pieces. The neutron is an isolated system, so its momentum should be conserved. The observed decay products, the electron and proton, move in opposite directions.
Visualize:



Known
$m_e = 9.11 \times 10^{-31}$ kg
$m_P = 1.67 \times 10^{-27}$ kg
$v_e = 3.0 \times 10^7$ m/s
$v_P = -1.0 \times 10^5$ m/s

Find
magnitude and direction of neutrino's momentum

Solve: **(a)** The initial momentum is $p_{ix} = 0$ kg m/s. The final momentum $p_{fx} = m_e v_e + m_p v_p$ is

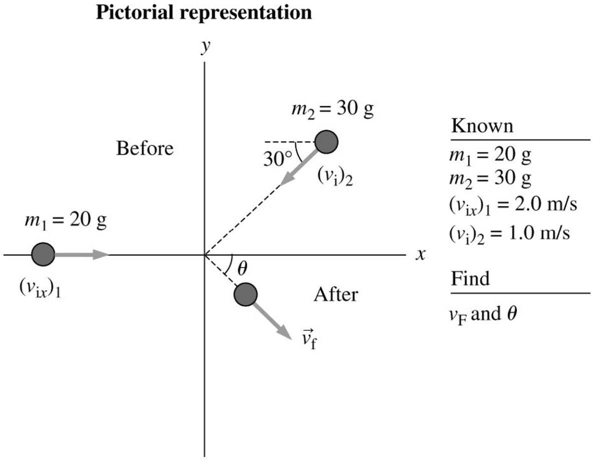
$$p_{fx} = 2.73\times10^{-23}\ \text{kg m/s} - 1.67\times10^{-22}\ \text{kg m/s} = -1.4\times10^{-22}\ \text{kg m/s}$$

No, momentum does not seem to be conserved.
(b) and **(c)** If the neutrino is needed to conserve momentum, then $p_e + p_P + p_{\text{neutrino}} = 0$ kg m/s. This requires

$$p_{\text{neutrino}} = -(p_e + p_P) = +1.4\times10^{-22}\ \text{kg m/s}$$

The neutrino must "carry away" 1.4×10^{-22} kg m/s of momentum in the same direction as the electron.

11.70. Model: Model the two balls of clay as particles. Our system comprises these two balls. Momentum is conserved in the perfectly inelastic collision.
Visualize:



Solve: Applying conservation of momentum in the x-direction gives

$$\begin{aligned} p_{fx} &= p_{ix} = m_1(v_{ix})_1 + m_2(v_{ix})_2 \\ &= (0.020\ \text{kg})(2.0\ \text{m/s}) - (0.030\ \text{kg})(1.0\ \text{m/s})\cos30° = 0.0140\ \text{kg m/s} \end{aligned}$$

The y-component of the final momentum is

$$\begin{aligned} p_{fy} &= p_{iy} = m_1(v_{iy})_1 + m_2(v_{iy})_2 \\ &= (0.02\ \text{kg})(0\ \text{m/s}) - (0.03\ \text{kg})(1.0\ \text{m/s})\sin30° = -0.0150\ \text{kg m/s} \end{aligned}$$

$$p_f = \sqrt{(0.014\ \text{kg m/s})^2 + (-0.015\ \text{kg m/s})^2} = 0.0205\ \text{kg m/s}$$

Since $p_f = (m_1 + m_2)v_f = 0.0205$ kg m/s, the final speed is

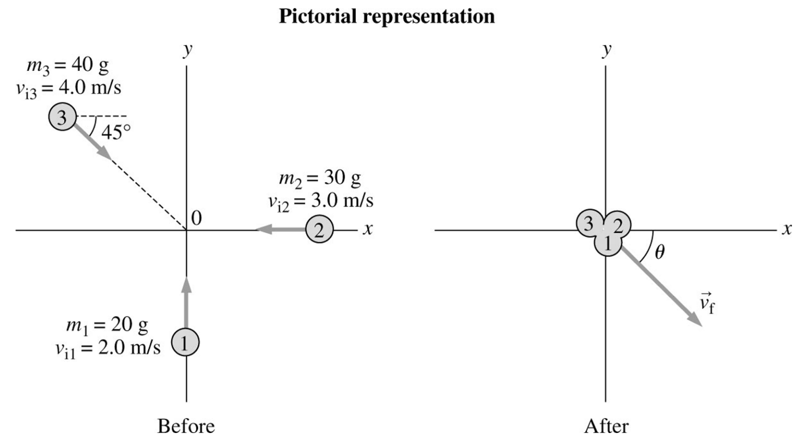
$$v_f = \frac{0.0205\ \text{kg m/s}}{(0.02 + 0.03)\ \text{kg}} = 0.41\ \text{m/s}$$

and the direction is

$$\theta = \tan^{-1}\frac{|p_{fy}|}{p_{fx}} = \tan^{-1}\frac{0.015}{0.014} = 47°\quad \text{south of east}$$

11.71. Model: Model the three balls of clay as particle 1 (moving north), particle 2 (moving west), and particle 3 (moving southeast). The three stick together during their collision, which is perfectly inelastic. The momentum of the system is conserved.

Visualize:



Solve: The three initial momenta are

$$\vec{p}_{i1} = m_1\vec{v}_{i1} = (0.020\text{ kg})(2.0\text{ m/s})\hat{j} = 0.040\hat{j}\ \text{ kg m/s}$$

$$\vec{p}_{i2} = m_2\vec{v}_{i2} = (0.030\text{ kg})(-3.0\text{ m/s }\hat{i}) = -0.090\hat{i}\ \text{kg m/s}$$

$$\vec{p}_{i3} = m_3\vec{v}_{i3} = (0.040\text{ kg})[(4.0\text{ m/s})\cos45°\hat{i} - (4.0\text{ m/s})\sin45°\hat{j}] = (0.113\hat{i} - 0.113\hat{j})\text{ kg m/s}$$

Since $\vec{p}_f = \vec{p}_i = \vec{p}_{i1} + \vec{p}_{i2} + \vec{p}_{i3}$, we have

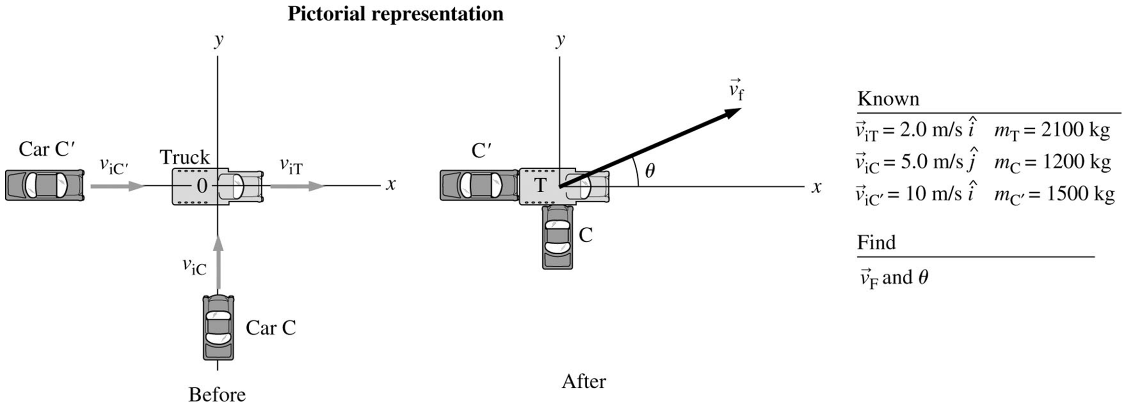
$$(m_1 + m_2 + m_3)\vec{v}_f = (0.023\hat{i} - 0.073\hat{j})\text{ kg m/s} \quad\Rightarrow\quad \vec{v}_f = (0.256\hat{i} - 0.811\hat{j})\text{ m/s}$$

$$v_f = \sqrt{(0.256\text{ m/s})^2 + (-0.811\text{ m/s})^2} = 0.85\text{ m/s}$$

$$\theta = \tan^{-1}\frac{|v_{fy}|}{v_{fx}} = \tan^{-1}\frac{0.811}{0.256} = 72°\ \text{ below the } x\text{-axis.}$$

11.72. Model: Model the truck (T) and the two cars (C and C′) as particles. The three forming our system stick together during their collision, which is perfectly inelastic. Since no significant external forces act on the system during the brief collision time, the momentum of the system is conserved.

Visualize:


Pictorial representation
y
Car C′
$v_{iC'}$
Truck
0
v_{iT}
x
v_{iC}
Car C
Before
y
$\vec{v}_f$
C′
T
θ
x
C
After
Known
$\vec{v}_{iT} = 2.0$ m/s $\hat{i}$ $m_T = 2100$ kg
$\vec{v}_{iC} = 5.0$ m/s $\hat{j}$ $m_C = 1200$ kg
$\vec{v}_{iC'} = 10$ m/s $\hat{i}$ $m_{C'} = 1500$ kg
Find
$\vec{v}_F$ and θ

Solve: The three momenta are

$$\vec{p}_{\text{iT}} = m_{\text{T}}\vec{v}_{\text{iT}} = (2100\text{ kg})(2.0\text{ m/s})\hat{i} = 4200\hat{i}\text{ kg m/s}$$

$$\vec{p}_{\text{iC}} = m_{\text{C}}\vec{v}_{\text{iC}} = (1200\text{ kg})(5.0\text{ m/s})\hat{j} = 6000\hat{j}\text{ kg m/s}$$

$$\vec{p}_{\text{iC}'} = m_{\text{C}'}\vec{v}_{\text{iC}'} = (1500\text{ kg})(10\text{ m/s})\hat{i} = 15{,}000\hat{i}\text{ kg m/s}$$

$$\vec{p}_{\text{f}} = \vec{p}_{\text{i}} = \vec{p}_{\text{iT}} + \vec{p}_{\text{iC}} + \vec{p}_{\text{iC}'} = (19{,}200\hat{i} + 6000\hat{j})\text{ kg m/s}$$

$$p_{\text{f}} = (m_{\text{T}} + m_{\text{C}} + m_{\text{C}'})v_{\text{f}} = \sqrt{(19{,}200\text{ kg m/s})^2 + (6000\text{ kg m/s})^2}$$

$$v_{\text{f}} = 4.2\text{ m/s},\ \theta = \tan^{-1}\frac{p_y}{p_x} = \tan^{-1}\frac{6000}{19{,}200} = 17^\circ\text{ above the }+x\text{-axis}$$

Assess: A speed of 4.2 m/s for the entangled three vehicles is reasonable since the individual speeds of the cars and the truck before entanglement were of the same order of magnitude.

11.73. Model: Ignore gravitational forces.

Visualize: We are given $m_{\text{R}} = 150\text{ kg}$, $v_{\text{ex}} = 2500\text{ m/s}$, $m_{\text{F0}} = 600\text{ kg}$. After 10 s 1/3 of the fuel is gone, so $m_{\text{F}} = 400\text{ kg}$, and after 20 s 2/3 of the fuel is gone so $m_{\text{F}} = 200\text{ kg}$.

Solve: We need the initial mass.

$$m_0 = m_{\text{R}} + m_{\text{F0}} = 150\text{ kg} + 600\text{ kg} = 750\text{ kg}$$

For $t = 10\text{ s}$:

$$m = m_{\text{R}} + m_{\text{F}} = 150\text{ kg} + 400\text{ kg} = 550\text{ kg}$$

$$v = v_{\text{ex}}\ln\left(\frac{m_0}{m}\right) = (2500\text{ m/s})\ln\left(\frac{750\text{ kg}}{550\text{ kg}}\right) = 780\text{ m/s}$$

For $t = 20\text{ s}$:

$$m = m_{\text{R}} + m_{\text{F}} = 150\text{ kg} + 200\text{ kg} = 350\text{ kg}$$

$$v = v_{\text{ex}}\ln\left(\frac{m_0}{m}\right) = (2500\text{ m/s})\ln\left(\frac{750\text{ kg}}{350\text{ kg}}\right) = 1900\text{ m/s}$$

For $t = 30\text{ s}$ the fuel is gone:

$$m = m_{\text{R}} + m_{\text{F}} = 150\text{ kg} + 0\text{ kg} = 150\text{ kg}$$

$$v = v_{\text{ex}}\ln\left(\frac{m_0}{m}\right) = (2500\text{ m/s})\ln\left(\frac{750\text{ kg}}{150\text{ kg}}\right) = 4000\text{ m/s}$$

This last value is v_{max}.

Assess: The rate the speed increases gets bigger as the mass decreases.

11.74. Model: Ignore gravitational forces.

Visualize: We are given $v_{\text{ex}} = 2000\text{ m/s}$.

Solve: (a) We are given for the single stage $m_{\text{R}} = 200\text{ kg},\ \text{m}_{\text{F0}} = 800\text{ kg}$.

$$v_{\text{max}} = v_{\text{ex}}\ln\left(\frac{m_{\text{R}} + m_{\text{F0}}}{m_{\text{R}}}\right) = (2000\text{ m/s})\ln\left(\frac{200\text{ kg} + 800\text{ kg}}{200\text{ kg}}\right) = 3200\text{ m/s}$$

(b) Now we repeat this in two stages: $m_R = 100$ kg, $m_{F0} = 400$ kg for *each* stage. First find $v_{\max}$ for the first stage which starts with all the fuel and all of the mass of both rocket stages, but the final mass only has half the fuel, but still both rocket stages.

$$\text{1st stage: } v_{\max} = v_{\text{ex}} \ln\left(\frac{2m_R + 2m_{F0}}{2m_R + m_{F0}}\right) = (2000 \text{ m/s})\ln\left(\frac{200 \text{ kg} + 800 \text{ kg}}{200 \text{ kg} + 400 \text{ kg}}\right) = 1020 \text{ m/s}$$

For the second stage we don't assume the initial speed is zero.

$$\int_{v_0}^{v} dv_x = -v_{\text{ex}} \int_{m_0}^{m} \frac{dm}{m}$$

$$v - v_0 = -v_{\text{ex}} \ln\left(\frac{m}{m_0}\right)$$

$$v = v_0 + v_{\text{ex}} \ln\left(\frac{m_0}{m}\right)$$

Now find the maximum speed.

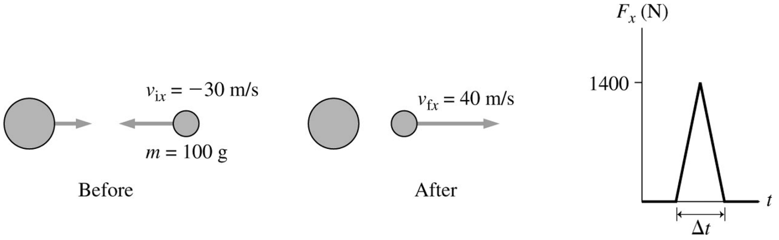
$$\text{2nd stage: } v_{\max} = v_0 + v_{\text{ex}} \ln\left(\frac{m_R + m_{F0}}{m_R}\right)$$

$$= 1020 \text{ m/s} + (2000 \text{ m/s})\ln\left(\frac{100 \text{ kg} + 400 \text{ kg}}{100 \text{ kg}}\right) = 4200 \text{ m/s}$$

Assess: The speed after the first stage of the two-stage rocket is not yet half of the maximum speed of the one-stage rocket, but the second stage, with its smaller mass, is able to achieve a higher final speed.

11.75. (a) A 100 g ball traveling to the left at 30 m/s is batted back to the right at 40 m/s. The force curve for the force of the bat on the ball can be modeled as a triangle with a maximum force of 1400 N. How long is the ball in contact with the bat?

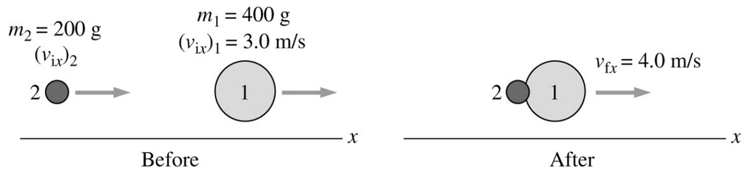
(b)



(c) The solution is $\Delta t = 0.100 \text{ s} = 10$ ms.

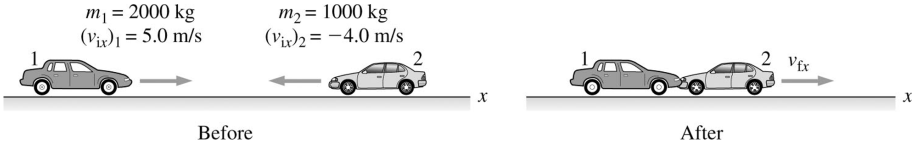
11.76. (a) A 200 g ball of clay traveling to the right overtakes and collides with a 400 g ball of clay traveling to the right at 3.0 m/s. The balls stick and move forward at 4.0 m/s. What was the speed of the 200 g ball of clay?

(b)



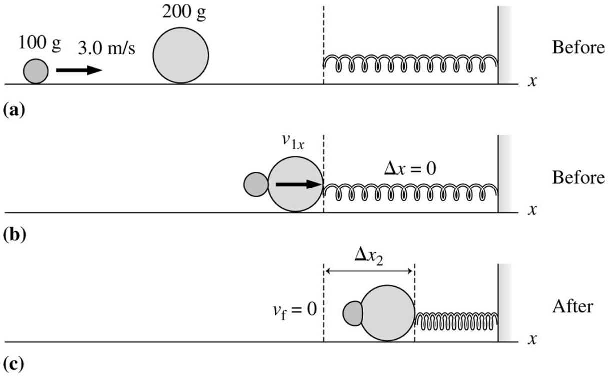
(c) The solution is $(v_{ix})_2 = 6.0$ m/s.

11.77. (a) A 2000 kg auto traveling east at 5.0 m/s suffers a head-on collision with a small 1000 kg auto traveling west at 4.0 m/s. They lock bumpers and stick together after the collision. What will be the speed and direction of the combined wreckage after the collision?
(b)



(c) The solution is $v_{fx} = 2.0$ m/s along the $+x$ direction.

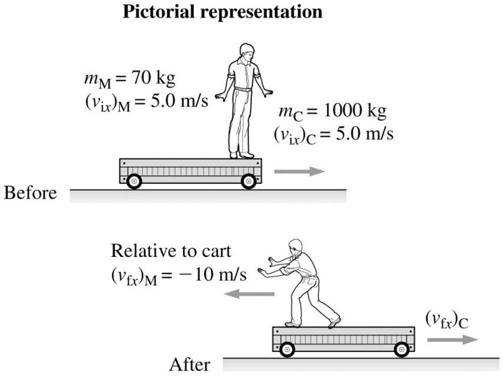
11.78. (a) A 100 g lump of clay traveling at 3.0 m/s strikes and sticks to a 200 g lump of clay at rest on a frictionless surface. The combined lumps smash into a horizontal spring with $k = 3.0$ N/m. The other end of the spring is firmly anchored to a fixed post on the surface. How far will the spring compress?
(b)



(c) Solving the conservation of momentum equation we get $v_{1x} = 1.0$ m/s. Substituting this value into the conservation of energy equation yields $\Delta x_2 = 32$ cm.

Challenge Problems

11.79. Model: The cart + man (C + M) is our system. It is an isolated system, and momentum is conserved.
Visualize:



Solve: The conservation of momentum equation $p_{\text{fx}} = p_{\text{ix}}$ gives

$$m_{\text{M}}(v_{\text{fx}})_{\text{M}} + m_{\text{C}}(v_{\text{fx}})_{\text{C}} = m_{\text{M}}(v_{\text{ix}})_{\text{M}} + m_{\text{C}}(v_{\text{ix}})_{\text{C}}$$

Note that $(v_{\text{fx}})_{\text{M}}$ and $(v_{\text{fx}})_{\text{C}}$ are the final velocities of the man and the cart relative to the ground. What is given in this problem is the velocity of the man relative to the moving cart. The man's velocity relative to the ground is

$$(v_{\text{fx}})_{\text{M}} = (v_{\text{fx}})_{\text{C}} - 10 \text{ m/s}$$

With this form for $(v_{\text{fx}})_{\text{M}}$, we rewrite the momentum conservation equation as

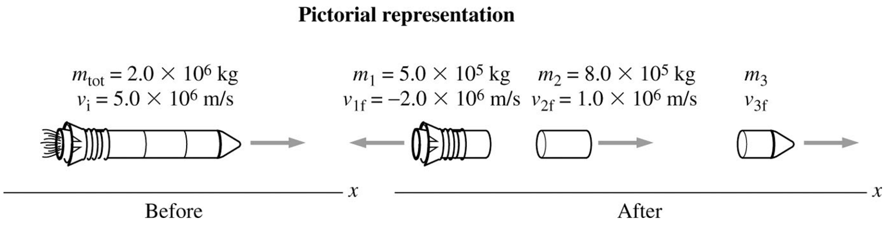
$$m_{\text{M}}[(v_{\text{fx}})_{\text{C}} - 10 \text{ m/s}] + m_{\text{C}}(v_{\text{fx}})_{\text{C}} = m_{\text{M}}(5.0 \text{ m/s}) + m_{\text{C}}(5.0 \text{ m/s})$$

$$(70 \text{ kg})[(v_{\text{fx}})_{\text{C}} - 10 \text{ m/s}] + (1000 \text{ kg})(v_{\text{fx}})_{\text{C}} = (1000 \text{ kg} + 70 \text{ kg})(5.0 \text{ m/s})$$

$$(v_{\text{fx}})_{\text{C}}[1000 \text{ kg} + 70 \text{ kg}] = (1070 \text{ kg})(5.0 \text{ m/s}) + (70 \text{ kg})(10 \text{ m/s}) \quad \Rightarrow \quad (v_{\text{fx}})_{\text{C}} = 5.7 \text{ m/s}$$

11.80. Model: This is an isolated system, so momentum is conserved in the explosion. Momentum is a *vector* quantity, so the direction of the initial velocity vector $\vec{v}_1$ establishes the direction of the momentum vector. The final momentum vector, after the explosion, must still point in the +*x*-direction. The two known pieces continue to move along this line and have no *y*-components of momentum. The missing third piece cannot have a *y*-component of momentum if momentum is to be conserved, so it must move along the *x*-axis—either straight forward or straight backward. We can use conservation laws to find out.

Visualize:


Pictorial representation
$m_{\text{tot}} = 2.0 \times 10^6$ kg
$v_{\text{i}} = 5.0 \times 10^6$ m/s
$m_1 = 5.0 \times 10^5$ kg
$v_{1\text{f}} = -2.0 \times 10^6$ m/s
$m_2 = 8.0 \times 10^5$ kg
$v_{2\text{f}} = 1.0 \times 10^6$ m/s
m_3
$v_{3\text{f}}$
x
Before
x
After

Solve: From the conservation of mass, the mass of piece 3 is

$$m_3 = m_{\text{total}} - m_1 - m_2 = 7.0 \times 10^5 \text{ kg}$$

To conserve momentum along the *x*-axis, we require

$$p_{\text{i}} = m_{\text{total}} v_{\text{i}} = p_{\text{f}} = p_{1\text{f}} + p_{2\text{f}} + p_{3\text{f}} = m_1 v_{1\text{f}} + m_2 v_{2\text{f}} + p_{3\text{f}}$$

$$p_{3\text{f}} = m_{\text{total}} v_{\text{i}} - m_1 v_{1\text{f}} - m_2 v_{2\text{f}} = +1.02 \times 10^{13} \text{ kg m/s}$$

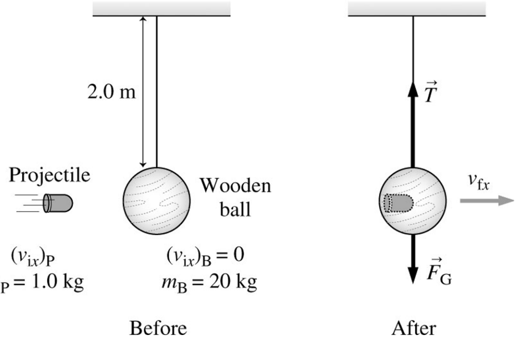
Because $p_{3\text{f}} > 0$, the third piece moves in the $+x$-direction, that is, straight forward. Because we know the mass m_3, we can find the velocity of the third piece as follows:

$$v_{3\text{f}} = \frac{p_{3\text{f}}}{m_3} = \frac{1.02 \times 10^{13} \text{ kg m/s}}{7.0 \times 10^5 \text{ kg}} = 1.5 \times 10^7 \text{ m/s}$$

The third piece moves to the right with a speed of 1.5×10^7 m/s.

11.81. Model: The projectile + wood ball are our system. In the collision, momentum is conserved.
Visualize:

Pictorial representation



Solve: The momentum conservation equation $p_{fx} = p_{ix}$ is

$$(m_P + m_B)v_{fx} = m_P(v_{ix})_P + m_B(v_{ix})_B \quad \Rightarrow \quad (1.0 \text{ kg} + 20 \text{ kg})v_{fx} = (1.0 \text{ kg})(v_{ix})_P + 0 \text{ kg m/s}$$

$$(v_{ix})_P = 21v_{fx}$$

We therefore need to determine v_{fx}. Newton's second law for circular motion is

$$T - F_G = T - (m_P + m_B)g = \frac{(m_P + m_B)v_{fx}^2}{r}$$

Using $T_{max} = 400$ N, this equation gives

$$400 \text{ N} - (1.0 \text{ kg} + 20 \text{ kg})(9.8 \text{ m/s}) = \frac{(1.0 \text{ kg} + 20 \text{ kg})v_{fx}^2}{2.0 \text{ m}} \quad \Rightarrow \quad (v_{fx})_{max} = 4.3 \text{ m/s}$$

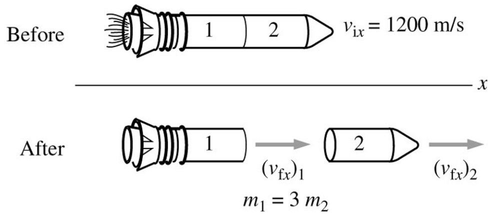
Going back to the momentum conservation equation,

$$(v_{ix})_P = 21v_{fx} = (21)(4.3 \text{ m/s}) = 90 \text{ m/s}$$

That is, the largest speed this projectile can have without causing the cable to break is 90 m/s.

11.82. Model: This is an "explosion" problem and momentum is conserved. The two-stage rocket is our system.
Visualize:

Pictorial representation



Solve: Relative to the ground, the conservation of momentum equation $p_{fx} = p_{ix}$ gives

$$m_1(v_{fx})_1 + m_2(v_{fx})_2 = (m_1 + m_2)v_{1x}$$

$$3 \text{ m}_2(v_{fx})_1 + m_2(v_{fx})_2 = (4 \text{ m}_2)(1200 \text{ m/s}) \quad \Rightarrow \quad 3(v_{fx})_1 + (v_{fx})_2 = 4800 \text{ kg m/s}$$

The fact that the first stage is pushed backward at 35 m/s relative to the second can be written

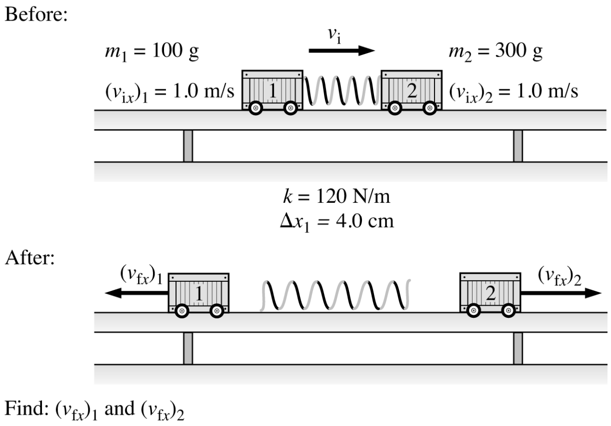
$$(v_{fx})_1 = -35 \text{ m/s} + (v_{fx})_2$$

Substituting this form of $(v_{fx})_1$ in the conservation of momentum equation,

$$3[-35\text{ m/s}+(v_{fx})_2]+(v_{fx})_2 = 4800\text{ kg m/s} \quad\Rightarrow\quad (v_{fx})_2 = 1.2\text{ km/s}$$

11.83. Model: Assume the spring is ideal and obeys Hooke's law. There's no friction. Mechanical energy $K+U_{sp}$ is conserved. Because the blocks and spring form an isolated system, total momentum is conserved.

Visualize:



Solve: This problem is most easily done by first solving it in a reference frame where the two carts are not initially moving (a reference frame moving at 1.0 m/s to the right relative to the laboratory) and then transforming into a reference frame where the initial velocities are 1.0 m/s. In the original frame (call it S) the initial kinetic energy is zero and the momentum is zero. Call this reference frame S in which the two carts are initially at rest, then transform the answer to the frame S′ in which the carts are initially moving to the right.

Thus in the S frame,

$$K_f + U_{Sp\,f} = K_i + U_{Sp\,i}$$

$$\tfrac{1}{2}m_1(v_f)_1^2 + \tfrac{1}{2}m_2(v_f)_2^2 + 0 = 0 + 0 + \tfrac{1}{2}k(\Delta x_i)^2$$

Use conservation of momentum in the S frame where the initial momentum is zero.

$$m_1(v_{fx})_1 + m_2(v_{fx})_2 = 0 \Rightarrow (v_{fx})_1 = -\left(\frac{m_2}{m_1}\right)(v_{fx})_2$$

The minus sign indicates that in the S frame the blocks move in opposite directions. Substitute into the energy equation.

$$\tfrac{1}{2}m_1\left(\tfrac{m_2}{m_1}(v_f)_2\right) + \tfrac{1}{2}m_2\left(v_f\right)_2^2 = \tfrac{1}{2}k(\Delta x_i)^2 \Rightarrow m_2\left(1+\tfrac{m_2}{m_1}\right)\left(v_f\right)_2^2 = k(\Delta x_i)^2$$

Solving for $(v_f)_2$, we find

$$(v_{fx})_2 = \sqrt{\frac{k(\Delta x_i)^2}{m_2(1+{}^{m_2}\!/_{m_1})}}$$

$$(v_{fx})_1 = -\frac{m_2}{m_1}(v_{fx})_2$$

Let the 100 g cart be Cart 1 and the 300 g cart be Cart 2. With $k = 120$ N/m and $\Delta x_i = 4.0$ cm,

$$(v_{fx})_2 = 0.40\text{ m/s}, \quad (v_{fx})_1 = -1.2\text{ m/s}$$

An object at rest in the S frame is traveling to the right at 1.0 m/s in the S′ frame. The equation of transformation is therefore

$$v_x' = v_x + 1.0 \text{ m/s}$$

In the S′ frame, the velocities of the carts are

$$(v_{\text{f}x})_1' = -1.2 \text{ m/s} + 1.0 \text{ m/s} = -0.2 \text{ m/s}$$

$$(v_{\text{f}x})_2' = 0.40 \text{ m/s} + 1.0 \text{ m/s} = 1.4 \text{ m/s}$$

Thus, the final velocity of the 100 g cart is 0.2 m/s to the left and the final velocity of the 200 g cart is 1.4 m/s to the right.

Assess: This problem could also be solved with more difficulty in the stationary reference frame (where the carts are initially moving) without the reference frame transformation.

11.84. Model: Do not ignore gravitational forces, but do ignore air resistance.

Visualize: We are given $v_{\text{ex}} = 2000$ m/s.

Solve: (a) Return to the original momentum equation in the rocket propulsion section, but now $p_{\text{f}y} \neq p_{\text{i}y}$ because the gravitational force exerts an impulse of $mg\Delta t$. It is still true that $P_{\text{f}y} = m_{\text{rocket}}(v_y)_{\text{rocket}} + m_{\text{fuel}}(v_y)_{\text{fuel}}$ and $P_{\text{i}y} = mv_y$.

$$p_{\text{f}y} = p_{\text{i}y} + J_y$$

$$(m + dm)(v_y + dv_y) + (-dm)(v_y - v_{\text{ex}}) = mv_y - mgdt$$

Cancel some terms and ignore the infinitesimal squared term.

$$mdv_y = -v_{\text{ex}}dm - mgdt$$

Divide by *m* and express *dt* in terms of *dm*: $R = |dm/dt|$ (which we assume to be constant).

$$dv_y = -v_{\text{ex}}\frac{dm}{m} - \frac{g}{R}dm$$

Integrate and assume the initial speed is zero.

$$\int_0^v dv_y = -v_{\text{ex}}\int_{m_0}^{m}\frac{dm}{m} - \frac{g}{R}\int_{m_0}^{m} dm$$

$$v = -v_{\text{ex}}\ln\left(\frac{m_0}{m}\right) - \frac{g}{R}(m_0 - m)$$

Just as before, w still see that when $m = m_0$, $v = 0$. Recall $m_0 = m_{\text{R}} + m_{\text{F0}}$. The maximum speed occurs when the fuel is completely gone and $m = m_{\text{R}}$.

$$v_{\max} = v_{\text{ex}}\ln\left(\frac{m_{\text{R}} + m_{\text{F0}}}{m_{\text{R}}}\right) - \frac{g}{R}(m_0 - m_{\text{R}}) = v_{\text{ex}}\ln\left(\frac{m_{\text{R}} + m_{\text{F0}}}{m_{\text{R}}}\right) - \frac{g}{R}(m_{\text{F0}})$$

This result seems to depend on the right variables in the right way.

(b) Assume $m_{\text{R}} = 330{,}000 \text{ kg} - 280{,}000 \text{ kg} = 50000 \text{ kg}$.

$$v_{\text{ex}} = \frac{F_{\text{thrust}}}{R} = \frac{4.1 \text{ MN}}{280{,}000 \text{ kg}/250 \text{ s}} = 3660 \text{ m/s}$$

For deep space:

$$v_{\max} = v_{\text{ex}}\ln\left(\frac{m_{\text{R}} + m_{\text{F0}}}{m_{\text{R}}}\right) = (3660 \text{ m/s})\ln\left(\frac{330{,}000 \text{ kg}}{50000 \text{ kg}}\right) = 6900 \text{ m/s}$$

For launching from the earth:

$$v_{\text{max}} = v_{\text{ex}} \ln\left(\frac{m_{\text{R}} + m_{\text{F0}}}{m_{\text{R}}}\right) - \frac{g}{R}(m_{\text{F0}})$$

$$= (3660\ \text{m/s})\ln\left(\frac{330{,}000\ \text{kg}}{50000\ \text{kg}}\right) - \frac{9.8\ \text{m/s}^2}{280{,}000\ \text{kg}/250\ \text{s}}(280{,}000\ \text{kg}) = 4500\ \text{m/s}$$

Assess: Launching from earth achieves a significantly lower final speed than doing so in deep space.